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Fuzzy adaptive H_∞ control for a class of nonlinear systems*

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Abstract. Combining both kinds of fuzzy logic forms including fuzzy T-S model and adaptive fuzzy logic systems, this paper presents an adaptive control scheme for a class of nonlinear systems. Firstly, the fuzzy T-S model is used to approximate the nonlinear systems, and the fuzzy control law of the fuzzy model is derived by the linear matrix inequality. Secondly, the adaptive fuzzy logic systems are constructed, and the modeling errors are eliminated by a compensator based on the adaptive fuzzy logic systems with three adjustable parameters: weights, centers and widths. It is proved that the closed loop system satisfies the anticipant performance. The simulation results demonstrate that the control scheme is effective.

Keywords: fuzzy T-S model, adaptive fuzzy logic systems, nonlinear systems

1 Introduction

The H_∞ control problem of nonlinear systems based on fuzzy T-S model has attracted considerable attention, mainly because the fuzzy T-S model provides an effective approach to linearize nonlinear systems .And this approach has been successful applied in [2, 3] . The stability problem of fuzzy control system is discussed in [9]. And the robust problem of fuzzy control system is researched in [5].However, the modeling error is neglected in these studies. Therefore, the designed controller does not always guarantee the stability of the original system. Besides, in nonlinear modeling, the paper[7] considers that the modeling error has the upper bound, the papers [1, 6] consider that the modeling error satisfies the matching condition, However, the upper bound and the matching condition are difficult to find in practice.

The adaptive fuzzy logic systems have the universal approximation property and could uniformly approximate nonlinear continuous functions to an arbitrary accuracy. The adaptive fuzzy logic systems could sufficiently make use of the linguistic information and the expert information. The adaptive fuzzy logic systems are used to model nonlinear systems by a set of fuzzy "if-then" rules. When a proper control is given to the model, an anticipant output is produced from the nonlinear systems. At present, the adaptive fuzzy logic systems have been successfully used in nonlinear control[4, 8].

Combining fuzzy T-S model and adaptive fuzzy logic systems, this paper presents a new H_∞ control scheme for a class of nonlinear systems. Firstly, the fuzzy T-S model is used to approximate the nonlinear systems, and the fuzzy controller is designed by the linear matrix inequalities to guarantee the stability of the fuzzy system. Secondly, the adaptive fuzzy logic systems are constructed, and the modeling error is eliminated by a compensator based on the adaptive fuzzy logic systems with three adjustable parameters: weights, centers and widths. It is proved that the closed-loop system satisfies the anticipant H_∞ performance. The simulation results demonstrate that the control scheme is effective.

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2 Problem formulation

Consider the following nonlinear system

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = x_3 \\ \cdots \\ \dot{x_n} = f(x_1, x_2, \cdots, x_n, u) + d' \end{cases} \tag{1}$$

where $x_1, x_2, \cdots, x_n \in R^n$ are the measurable state vectors, $u \in R^n$ is the control input vector, f are the nonlinear vector function, d' denotes the external disturbance. Denote $x = [x_1^T, x_2^T, \cdots, x_n^T]^T \in R^{n^2}$, $d = [0, 0, \cdots, 0, d'^T]^T$.

The system equation (1) can be approximated by a fuzzy T-S model composed of rules. The L rule of fuzzy model is in the following form. If $z_1(t)$ is F_1^i and, $\cdots$, and $z_s(t)$ is F_s^i,Then

$$\dot{x_t} = A_i x(t) + B_i u(t) + d, \quad i = 1, 2, \cdots, L \tag{2}$$

where $z_1(t), z_2(t), \cdots, z_s(t)$ are the premise variables, F_j^i (for $j = 1, 2, \cdots, s$) are the fuzzy sets, L is the number of if-then rules, A_i, B_i are some constant matrices with compatible dimensions.

$B_i = [0, 0, \cdots, 0, b_i^T]^T \in R^{n^2 \times n}$, $b_i \in R^{n \times n}$. The final output of the fuzzy system is inferred as follows

$$\dot{x_t} = \sum_{i=1}^{L} \mu_i A_i x(t) + \sum_{i=1}^{L} \mu_i B_i u(t) + d \tag{3}$$

where $\mu_i = \nu_i(z(t)) big/ \sum_{i=1}^{L} \nu_i(z(t))$, $\nu_i(z(t)) = \prod_j = 1^s F_j^i(z_j(t))$, $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i. It is easy to find that $\mu_i \geq 0$ for $i = 1, 2 \cdots, L$ and $\sum_{i=1}^{L} \mu_i = 1$ for all t . Therefore, the modeling error of the nonlinear system equation (1) can be expressed as

$$B\Delta(x) = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \Delta f \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \cdots \\ x_n \\ f \end{bmatrix} - \sum_{i=1}^{L} \mu_i A_i x(t) - \sum_{i=1}^{L} \mu_i B_i u(t) \tag{4}$$

where $B = \begin{bmatrix} 0 & 0 & \cdots & 0 & I_{n \times n} \end{bmatrix}^T$, $\triangle(x) = \begin{bmatrix} 0 & 0 & \cdots & 0 & \triangle f^T \end{bmatrix}^T$. Therefore, the nonlinear system equation (1) could be rearranged as

$$\dot{x_t} = \sum_{i=1}^{L} \mu_i A_i x(t) + \sum_{i=1}^{L} \mu_i B_i u(t) + B \triangle (x) + d \tag{5}$$

3 The design of the controller

If we do not consider the effect of the $\triangle(x)$. Denote $\triangle(x)$ in equation (5). Design a controller to guarantee to stabilize the corresponding closed loop system of the linear part of the nonlinear system equation (5), and make the H_∞ performance satisfied.

The fuzzy state feedback controller is in the following form:
if $z_1(t)$ is F_1^i and, $\cdots$, and $z_s(t)$ is F_s^i, then

$$u(t) = K_i x(t), \quad i = 1, 2, \cdots, L \tag{6}$$

where K_i are matrixes with proper dimensions.

The overall state feedback controller is given by

$$u(t) = \sum_{i=1}^{L} \mu_i K_i x(t) \tag{7}$$

Because of the existence of the fuzzy modeling error, it is difficult to make the system equation (1) satisfy the anticipant H_∞ performance by only using the fuzzy state feedback controller. In this situation, we construct the fuzzy logic systems to eliminate the fuzzy modeling error. Therefore, the nonlinear system is stabilized and the desired H_∞ performance is achieved.

Combining the fuzzy state feedback control with the fuzzy-logic-system-based control, we design the control law

$$u(t) = u_l(t) - u_f(t) \tag{8}$$

where $u_l(t)$ denotes the fuzzy control law in equation (6), $u_f(t)$ is the compensator based on the adaptive fuzzy logic systems.

Choose the adaptive compensator

$$u_f(t) = E^{-1}\hat{u}(x|\Theta, \alpha, \delta) \tag{9}$$

$$E^T(I + EE^T)^{-1}\hat{u}(x|\Theta, \alpha, \delta) \tag{10}$$

If E is nonsingular, equation (9) is chosen, or else, equation (10) is chosen. Where $\hat{u}(x|\Theta, \alpha, \delta)$ is constructed by the adaptive fuzzy logic systems, $E_i = b_i \in R^{n\times n}$, $E = \sum_{i=1}^{L} \mu_i b_i$.

Denote $\bar{B} = -B$. Substituting equation (8) into equation(5), we get a closed-loop system

$$\dot{x_t} = \sum_{i=1}^{L}\sum_{j=1}^{L} \mu_i\mu_j \bar{A}_{ij} x(t) + d + (\hat{u}(x|\Theta, \alpha, \delta) - \triangle(x)) \tag{11}$$

where $\bar{A}_{ij} = A_i + B_i K_j$ $(i, j = 1, \cdots, L)$.

If the fuzzy logic systems $\hat{u}(x|\Theta, \alpha, \delta)$ could eliminate $\triangle(x)$, then the closed-loop system equation (11) is stable.

It has been proven that the adaptive fuzzy logic systems have the universal approximation property. We construct the adaptive fuzzy logic systems to approximate the vector function $\triangle(x)$ with n dimensions. The fuzzy logic systems have the form $\hat{\triangle}(x|\Theta, \alpha, \delta) = \Psi(x, \alpha, \delta)\Theta$, where the fuzzy basis function $\Psi(x, \alpha, \delta) = diag[\xi_1(x, \alpha_1, \delta_1), \cdots, \xi_n(x, \alpha_n, \delta_n)]$, $\Theta = [\theta_1^T, \theta_2^T, \cdots, \theta_n^T,]$, $\alpha = diag[\alpha_1, \cdots, \alpha_n]$, $\delta = diag[\delta_1, \cdots, \delta_n]$, θ_i (for $i = 1, 2, \cdots, n$) are the column vectors, α_i, δ_i (for $i = 1, 2, \cdots, n$) are the row vectors. The weights Θ , the centers α and the widths are the adjustable parameters. We have the following theorem:

Theorem 1. *Define the estimation errors of the parameter vector and the parameter matrix* $\tilde{\Theta} = \Theta - \Theta^* = (\theta_i - \theta_i^*)_{n\times 1}$, $\tilde{\alpha} = \alpha - \alpha^* = diag(\alpha_1 - \alpha_1^*, \cdots, \alpha_n - \alpha_n^*)$, $\tilde{\delta} = diag(\delta_1 - \delta_1^*, \cdots, \delta_n - \delta_n^*)$, $\tilde{\xi}_i = \xi_i(x, \alpha_i, \delta_i) - \xi_i^*(x, \alpha_i, \delta_i)$ *(for* $i = 1, 2, \cdots, n$*), then the approximation error for the vector function* $\Delta(x) = [\Delta_1(x), \cdots, \Delta_n(x)]^T$ *can be expressed as*

$$\hat{\Delta}(x|\Theta, \alpha, \delta) - \Delta(x) = (\Psi(x) - \alpha\Psi_\alpha(x) - \delta\Psi_\delta(x))\tilde{\Theta} + (\tilde{\alpha}\Psi_\alpha(x) + \tilde{\delta}\Psi_\delta(x))\Theta + w_1$$

where $\Psi(x) = \Psi(x, \alpha, \delta), \Psi_\alpha(x) = diag[\xi_{1\alpha_1}(x, \alpha_1, \delta_1), \cdots, \xi_{n\alpha_n}(x, \alpha_n, \delta_n)]$,

$\Psi_\delta(x) = diag[\xi_{1\delta_1}(x, \alpha_1, \delta_1), \cdots, \xi_{n\delta_n}(x, \alpha_n, \delta_n)]$,

$\xi_{i\alpha_i}(x, \alpha_i, \delta_i)$ *and* $\xi_{i\delta_i}(x, \alpha_i, \delta_i)$ *denote partial derivatives of* ξ_i *with* α_i *and* δ_i *,respectively.* w_1 *is the residual term.*

Proof. Denote $(\tilde{\Delta}_i)_{n\times 1} = \tilde{\Delta}(x|\Theta_1, \alpha, \delta) - \Delta(x)$

$$\tilde{\Delta}_i = \tilde{\Delta}_i(x|\theta_i, \alpha_i, \delta_i) - \Delta_i(x) = \xi_i(x, \alpha_i, \delta_i)\tilde{\theta}_i + \tilde{\xi}_i\theta_i - \tilde{\xi}_i\theta_i^* + \bar{\varepsilon}_i \tag{12}$$

where $\bar{\varepsilon}_i = \tilde{\Delta}_i(x|\theta_i^*, \alpha_i^*, \delta_i^*) - \Delta_i(x)$.
Using the Taylor expansion of $\xi_i(x, \alpha_i, \delta_i)$ in (α_i^*, δ_i^*) ,we derive

$$\begin{aligned}\xi_i(x, \alpha_i, \delta_i) &= \xi_i(x, \alpha_i^*, \delta_i^*) + (\alpha_i - \alpha_i^*)\xi_{i\alpha} + (\delta_i - \delta_i^*)\xi_{i\delta} + o(x, \alpha_i - \alpha_i^*, \delta_i - \delta_i^*) \\ &= \xi_i(x, \alpha_i^*, \delta_i^*) + \tilde{\alpha}_i\xi_{i\alpha} + \tilde{\delta}_i\xi_{i\delta} + o(x, \tilde{\alpha}_i, \tilde{\delta}_i)\end{aligned} \tag{13}$$

where $o(x, \tilde{\alpha}_i, \tilde{\delta}_i)$ denotes the high order argument.

Substituting equation (13) into equation (12) yields

$$\tilde{\Delta}_i = (\xi_i(x, \alpha_i, \delta_i) - \alpha_i\xi_{i\alpha_i} - \delta_i\xi_{i\delta_i})\tilde{\theta}_i + (\tilde{\alpha}_i\xi_{i\alpha_i} + \tilde{\delta}_i\xi_{i\delta_i})\theta_i + (\alpha_i^*\xi_{i\alpha_i} + \delta_i^*\xi_{i\delta_i})\tilde{\theta}_i + o(x, \tilde{\alpha}_i, \tilde{\delta}_i)\theta_i^* + \bar{\varepsilon}_i$$

Denote $w_1 = (w_1 i)_n \times 1$,$w_1 i = (\alpha_i^*\xi_{i\alpha_i} + \delta_i^*\xi_{i\delta_i})\tilde{\theta}_i + o(x, \tilde{\alpha}_i, \tilde{\delta}_i)\theta_i^* + \bar{\varepsilon}_i$.
We have

$$\hat{\Delta}(x|\Theta, \alpha, \delta) = (\Psi(x) - \alpha\Psi_\alpha(x) - \delta\Psi_\delta(x))\tilde{\Theta} + (\tilde{\alpha}\Psi_\alpha(x) + \tilde{\delta}\Psi_\delta(x))\Theta + w_1$$

The proof is completed.

Denote $\bar{w}_1 = \begin{bmatrix} 0 & 0 & \cdots & 0 & w_1^T \end{bmatrix}^T$, $\bar{w} = d - \bar{w}_1$. By use of Theorem 1, (11) is rearranged as

$$\dot{x}_t = \sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}x(t) + \bar{w} + \bar{B}[(\Psi(x) - \alpha\Psi_\alpha(x) - \delta\Psi_\delta(x))\tilde{\Theta} + (\tilde{\alpha}\Psi_\alpha(x) + \tilde{\delta}\Psi_\delta(x))\Theta] \tag{14}$$

If the following parameter updating laws are chosen as

$$\begin{aligned}&\dot{\Theta} = -\eta_1(\Psi(x) - \alpha\Psi_\alpha(x) - \delta\Psi_\delta(x))^T(\bar{B}^T Px) \\ &\dot{\alpha} = -\eta_2\bar{B}^T Px * (\Psi_\alpha(x)\Theta)^T \\ &\dot{\delta} = -\eta_3\bar{B}^T Px * (\Psi_\delta(x)\Theta)^T\end{aligned} \tag{15}$$

where $\eta_1, \eta_2 and \eta_3$ are positive constants. We have the following theorem:

Theorem 2. *For the nonlinear system equation (1), the control law is chosen as equation (8) composed of the fuzzy state feedback control law equation (6) and the compensator equation (9), (10) based on the adaptive fuzzy logic systems, and the parameter updating laws are chosen as equation (15), then the closed-loop system equation (11) satisfies the H_∞ performance*

$$\begin{aligned}\int_0^T x^T(t)Qx(t)dt \le x^T(0)Px(0) + \frac{1}{\eta_1}\tilde{\Theta}^T(0)\tilde{\Theta}(0) \\ +\frac{1}{\eta_2}tr(\tilde{\alpha}^T(0)\tilde{\alpha}(0)) + \frac{1}{\eta_3}tr(\tilde{\delta}^T(0)\tilde{\delta}(0)) + \rho^2\int_0^T(\bar{w}^T\bar{w})dt\end{aligned} \tag{16}$$

where $\rho > 0, P, Q$ are some symmetric and positive definite matrices. P and Q satisfy the following matrix inequalities

$$\bar{A_{ij}}^T P + P\bar{A_{ij}} + \frac{1}{\rho^2}PP + Q < 0 \quad (i, j = 1, \cdots, L) \tag{17}$$

Proof. Choose the Lyapunov function $V + \frac{1}{2}x^T(t)Px(t) + \frac{1}{2\eta_1}\tilde{\Theta}^T\tilde{\Theta} + \frac{1}{2\eta_2}tr(\tilde{\alpha}^T\tilde{\alpha}) + \frac{1}{2\eta_3}tr(\tilde{\delta}^T\tilde{\delta})$

$$\begin{aligned}\dot{V} &= \frac{1}{2}[\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)] + \frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}} + \frac{1}{\eta_2}tr(\tilde{\alpha}^T\dot{\tilde{\alpha}}) + \frac{1}{\eta_3}tr(\tilde{\delta}^T\dot{\tilde{\delta}}) \\ &= \frac{1}{2}[(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}x(t))^T Px(t) + x^T(t)P(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}x(t)) + \bar{w}^T Px(t) + x^T(t)P\bar{w}] \\ &\quad +[x^T P\bar{B}(\Psi(x) - \alpha\Psi_\alpha(x) - \delta\Psi_\delta(x))\tilde{\Theta} + \frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}}] + [x^T P\bar{B}\tilde{\alpha}\Psi_\alpha(x)\Theta + \frac{1}{\eta_2}tr(\tilde{\alpha}^T\dot{\tilde{\alpha}})] \\ &\quad +[x^T P\bar{B}\tilde{\delta}\Psi_\delta(x)\Theta + \frac{1}{\eta_3}tr(\tilde{\delta}^T\dot{\tilde{\delta}})]\end{aligned}$$

Because of $\dot{\tilde{\Theta}} = \dot{\Theta}, \dot{\tilde{\alpha}} = \dot{\alpha}, \dot{\tilde{\delta}} = \dot{\delta}$ using (15), we yield

$$\begin{aligned}
\dot{V} &= \frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[x^T(t)\bar{A}_{ij}^T Px(t)] + \frac{1}{2}\bar{w}^T Px(t) + \frac{1}{2}x^T(t)P\bar{w} \\
&\quad -\frac{1}{2}\rho^2\bar{w}^T\bar{w} - \frac{1}{2\rho^2}x^T(t)PPx(t) + \frac{1}{2}\rho^2\bar{w}^T\bar{w} + \frac{1}{2\rho^2}x^T(t)PPx(t) \\
&= \frac{1}{2}\{\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[x^T(t)\bar{A}_{ij}^T Px(t) + x^T(t)p\bar{A}_{ij}x(t)] \\
&\quad -(\frac{1}{\rho}Px(t) - \rho\bar{w})^T(\frac{1}{\rho}Px(t) - \rho\bar{w}) + \rho^2\bar{w}^2\bar{w} + \frac{1}{2\rho^2}x^T(t)PPx(t) \\
&\leq \frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j x^T(t)(\bar{A}_{ij}^T P + P\bar{A}_{ij} + \frac{1}{2\rho^2}PP)x(t) + \frac{1}{2}\rho^2\bar{w}^T\bar{w}
\end{aligned}$$

From equation (12), we derive $\dot{V} \leq -\frac{1}{2}x^T(t)Qx(t) + \frac{1}{2}\rho^2\bar{w}^T\bar{w}$.

Integrating the above equation from $t = 0$ to T yields equation (16). The proof is completed.

By the Schur complements, the inequalities equation (17) can be transformed into the linear matrix inequalities. Denote $W = P^{-1}$, $Y_j = K_jW$. The inequalities equation (17) are equivalent to the linear matrix inequalities

$$\begin{bmatrix} A_iW + WA_i^T + B_iY_j + (B_iY_j)^T + (\rho^2)^{-1}I & W \\ W & -Q^{-1} \end{bmatrix} < 0 \tag{18}$$

4 Simulation example

A 2-link manipulator system is used to illustrate the effectiveness of the proposed method

$$M(\theta)\ddot{\theta}(t) + C(\theta, \dot{\theta})\dot{\theta}(t) + G(\theta) = \tau(t) + d'$$

where $\theta = [\theta_1, \theta_2]^T$, d' is random noise.

Denote $x_1 = \theta$, $x_2 = \dot{\theta}$, $u = \tau$. The system is rearranged as the form equation (1): $\dot{x} = [x_2 \quad f(x,u) + d']^T$. Simulation results are shown in Fig. 1 and Fig. 2. The simulation results demonstrate

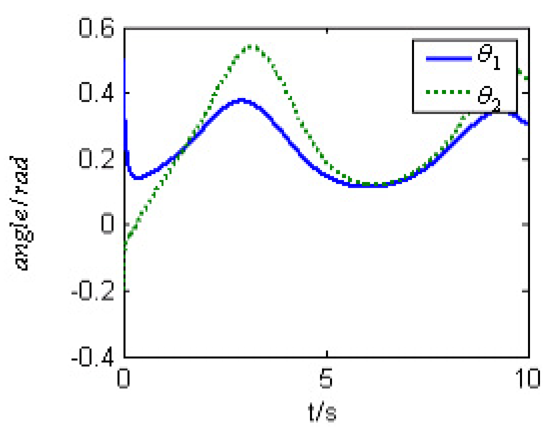

Fig. 1. The responses of the system states by only using fuzzy state feedback controller

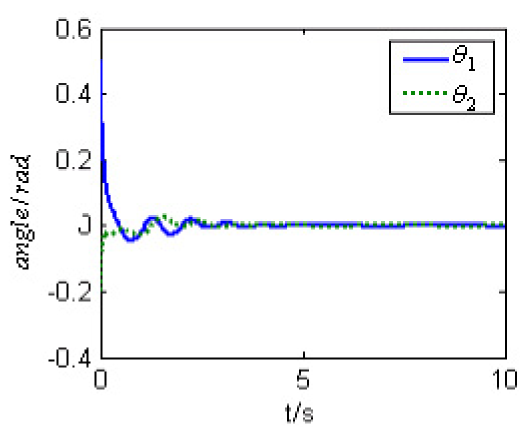

Fig. 2. The responses of the system states under fuzzy state feedback controller and adaptive compensator

that the proposed control scheme can guarantee to stabilize the nonlinear system rapidly.

5 Conclusion

This paper presents a control scheme for a class of nonlinear systems. The fuzzy T-S model is used to approximate the nonlinear systems. The modeling error is eliminated by a compensator based on the adaptive fuzzy logic systems. The simulation results demonstrate that the control scheme is effective.

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