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Bubble radius function-non isothermal film blowing*

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Abstract. In this paper, we present some precursor results on progressing into analytical solutions to the model for film blowing in a Nonisothermal situation. The model used here bears some refinement on Han and Park results for nonisothermal film blowing: correction in the model, better presentation, improved scaling, and ways of approaching the problem without neglecting gravity, all but the latter, as explained in our previous work for this same journal. The methods we use here are asymptotic expansions and Poincare series, partially inspired on the works of Battacharya et al.

Keywords: nonisothermal, film, blowing, newtonian, power-law, radius, solution, velocity, temperature

1 Introduction

Water vapor produces a thermal variation on the plastic. Therefore, studying things from a mathematical point of view might lead to success when dealing with situations like the one described in the works of Helen et al[4]. Taking into account the hazard that plastics are for the environment, any progress in the field would be more than welcome.

Han and Park[3] obtained some results on nonisothermal newtonian fluids as a restriction of power-law fluids. Tam[12] took a completely newtonian approach, his results dealing with the isothermal situation. Alaie and Papanastasiou[1] considered the nonisothermal situation of film blowing and treated it with an integral constitutive model. Kanai and White[5] produced an experimental study on the stability of nonisothermal (temperature dependent viscosity) film blowing of viscoelastic newtonian melts. Yamane and White[14] researched on the significance of non-newtonian viscosity on non-isothermal film blowing. M.R. Pinheiro[10] proposed some corrections on the model presented by Han and Park in order to more adequately deal with temperature changes. In this work, we depart from the corrected model[10] presented by M.R. Pinheiro, and we perform studies on the radius variations and its effects on the remaining functions involved in the system. For this work, we base ourselves partially on the knowledge contained in the works [8]-[13], [7], [9], [2], [4].

2 Notation

We tend to use capital letters for the non-scaled variables under study, which are also taken to vary independent from time, just dependent on height, that is, we study the system from a steady-state perspective, rather than from a dynamic one:

$R = R(Z) =$ Generic radius of the bubble, which varies with height;

$R_f =$ Radius of the bubble from freeze line upwards;

$E = E(Z) =$ Thickness of the bubble, which varies with height;

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$T = T(Z) =$ Temperature of the bubble, which varies with height;
$U = U(Z) =$ Meridional velocity of the fluid, which varies with height;
$H =$ Final thickness of the bubble, final being at the height of the frost line;
$Z_1 =$ Frost line height;
$F =$ Resulting force;
$Z =$ Height and small letters for the scaled variables under study;
$r = r(z) =$ radius;
$w = w(z) =$ thickness;
$s = s(z) =$ temperature;
$u = u(z) =$ velocity;
$z =$ height.

3 Force balance equation in the meridional direction

As explained in more detail in [10], our equation is:

$$F(Z) = F_o + \pi \Delta p(R^2 - R_o^2) + 2\pi\rho g \int_0^Z RE \frac{dZ}{\cos\theta} \tag{1}$$

But, it is also known that

$$F(Z) = T_{11} dA_1 \cos\theta = 2\pi RET_{11} \cos\theta \tag{2}$$

dA_1 being an infinitesimal part of the top of the fluid element under consideration. As it is stated in [10], T_{11} is part of our matrix of tensors, and θ is the angle formed between the Z axis and the velocity vector. F_o is the initial force.

4 Total composition of forces—all forces involved

Here, we proposed a small correction on the formula presented by Han and Park. For details, see [10]. Our pressure equation gets then translated into

$$\Delta P = \frac{P_L}{R_L} + \frac{P_H}{R_H} - \rho g E \sin\theta \tag{3}$$

where:

$$P_L = T_{11}E,\ P_H = T_{33}E,\ R_L = \frac{1}{\frac{|R''|}{(\cos\theta)^{-3}}},\ R_H = \frac{R}{\cos\theta} \tag{4}$$

where R'' stands for the unscaled second derivative of the radius function.

5 Material function—viscosity

According to Han and Park[3], one verifies the following viscosity relationship for a nonisothermal newtonian film blowing:

$$\eta_b = \eta_o e^{\frac{\left(\frac{1}{s(z)} - 1\right)E}{RT_o}} \tag{5}$$

6 Temperature equation—measurement of the temperature variations

We accept Han and Park's temperature equation provided in [3]:

$$s' = -Dr\sec\theta(s - s_a) - Yr\sec\theta(s^4 - s_a^4),$$

where

$$D = \frac{UR_o 2\pi Z_1}{Q\rho C_v} \tag{6}$$

and

$$Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda\epsilon}{Q\rho C_v} \tag{7}$$

7 Best scaling to provide us with the most complete set of initial values as possible

We decide to disagree with the scaling for r, s, and u, chosen by Han and Park because we believe that ours will provide us with a more complete set of initial conditions:

$$r(0) = 1, s(0) = 1, u(0) = 1, w(0) = 1 \tag{8}$$

Han and Park choose to scale their variables against a_o, the radius at the annular die. We chose to scale z against Z_1, the freeze line height. With this, $z = 1$ at the freeze line height. Our process, therefore, takes place between $z = 0$ and $z = 1$, a much nicer interval. We propose:

$$\eta = \frac{\eta_b}{\eta_o}; z = \frac{Z}{Z_1}; r = \frac{R}{R_o}; u = \frac{U}{U_o}; w = \frac{E}{E_o} \tag{9}$$

8 Our best system presentation for thermal effects on a newtonian film blowing situation

As a result from the previous function and equations, one gets to describe thermal effects on a newtonian film blowing situation through the system presented in [10]: for $z \in [0, a) \bigcup (d, 1]$, where a and d are the turning points of the bubble profile, we have, as an approximation, $r' = r'' = 0$, $r = k$:

$$\begin{cases} f_o + B(k^2 - 1) + \frac{A}{C}\int_0^Z (\lim_{u\to 0}\frac{1}{u})dz = 0, \\ s' = k[D(s - s_a) + Y(s^4 - s_a^4)]. \end{cases} \tag{10}$$

for $z \in (b, c)$, we have, as $r = c_3 z + c_4$ for some c_3, c_4 in $\Re$, $r' = c_3$ and $r'' = 0$. Therefore, our system is:

$$\begin{cases} 2B(c_3z + c_4)^3 u(1 + (Cc_3^2)^2 + AC(c_3z + c_4)^2 c_3(1 + (Cc_3)^2)^{\frac{3}{2}} \\ -C(1 + (Cc_3)^2)(2uc_3 + (c_3z + c_4)u')\eta = 0, \\ C(1 + C^2(c_3)^2)^{-1}\left(\frac{c_3u + 2u'(c_3z + c_4)}{u(c_3z + c_4)}\right) = f_o(\eta^{-1}) \\ +B((c_3z + c_4)^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_0^Z \frac{1}{u}(1 + C^2(c_3)^2)^{-\frac{1}{2}}dz, \\ s' = D(c_3z + c_4)((1 + C^2(c_3)^2)^{\frac{1}{2}})(s - s_a) + Y(c_3z + c_4)((1 + C^2(c_3)^2)^{\frac{1}{2}})(s^4 - s_a^4). \end{cases} \tag{11}$$

For $z \in [a,b]$ we have (since $r'' \geq 0$):

$$\begin{cases} 2Br^3u(1+(Cr'^2)^2 + ACr^2r'(1+(Cr')^2)^{\frac{3}{2}} \\ +C^3r''(2r^2u' + urr')\eta - C(1+(Cr')^2)(2ur' + ru')\eta = 0, \\ C(1+C^2(r')^2)^{-1}\left(\dfrac{r'u+2u'r}{ur}\right) = f_o(\eta^{-1}) + B(r^2-1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_o^Z \frac{1}{u}(1+C^2(r')^2)^{-\frac{1}{2}}dz, \\ s' = Dr((1+C^2(r')^2)^{\frac{1}{2}})(s-s_a) + Yr((1+C^2(r')^2))^{\frac{1}{2}}(s^4 - s_a^4). \end{cases} \tag{12}$$

whilst, for $z \in [c,d]$ we have (since $r'' \leq 0$):

$$\begin{cases} 2Br^3u(1+(Cr'^2)^2 + ACr^2r'(1+(Cr')^2)^{\frac{3}{2}} \\ -C^3r''(2r^2u' + urr')\eta - C(1+(Cr')^2)(2ur' + ru')\eta = 0, \\ C(1+C^2(r')^2)^{-1}\left(\dfrac{r'u+2u'r}{ur}\right) = f_o(\eta^{-1}) + B(r^2-1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_0^Z \frac{1}{u}(1+C^2(r')^2)^{-\frac{1}{2}}dz, \\ s' = Dr((1+C^2(r')^2)^{\frac{1}{2}})(s-s_a) + Yr((1+C^2(r')^2)^{\frac{1}{2}})(s^4 - s_a^4). \end{cases} \tag{13}$$

all with the following parameters, material function, and scaling:

$$\begin{aligned} &A = \frac{\rho g R_o^2}{\eta_o U_o}, B = \frac{\pi \Delta p R_o^3}{\eta_o Q}, C = \frac{R_o}{Z_1}, f_o = \frac{F_o R_o}{\eta_o Q}, \beta = \frac{E}{RT_o}, \\ &D = \frac{UR_o 2\pi Z_1}{Q\rho C_v}, Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda \epsilon}{Q\rho C_v}, \\ &\eta_b = \eta_o e^{\beta(\frac{1}{s(z)}-1)}, \eta = \frac{\eta_b}{\eta_o}, z = \frac{Z}{Z_1}, r = \frac{R}{R_o}, u = \frac{U}{U_o}, w = \frac{E}{E_o} \end{aligned} \tag{14}$$

And with the following initial conditions: $r(0) = w(0) = s(0) = u(0) = 1$.

9 Possible analytical solution

In comparing the system obtained from freeze line height to inflexion point, shown above, with the system previously obtained, we notice that the difference between the two of them relies on the term with r^3 for the first equation and on the term with f_o for the second, the difference just regarding signs.

We here use the fact that $u = r'$.

Consequently, we achieve the following set of equations:

for $z \in [0,a)\bigcup(b,c)\bigcup(d,1]$, where a and c are the turning points of the bubble profile, we have, as an approximation, $r' = r'' = 0, r = k$:

$$\begin{cases} f_o + B(k^2-1) + \frac{A}{C}\int_0^Z (\lim_{r'\to 0} \frac{1}{r'})dz = 0, \\ s' = k[D(s-s_a) + Y(s^4 - s_a^4)]. \end{cases} \tag{15}$$

For $z \in [a,b]$ we have (since $r'' \geq 0$):

$$\begin{cases} 2Br^3r'(1+(Cr'^2)^2 + ACr^2r'(1+(Cr')^2)^{\frac{3}{2}}) \\ +C^3r''(2r^2r''+r(r')^2)\eta - C(1+(Cr')^2)(2(r')^2+rr'')\eta = 0, \\ C(1+C^2(r')^2)^{-1}\left(\dfrac{(r')^2+2r''r}{r'r}\right) \\ = f_o(\eta^{-1}) + B(r^2-1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_0^Z \frac{1}{r'}(1+C^2(r')^2)^{-\frac{1}{2}}dz, \\ s' = Dr((1+C^2(r')^2)^{\frac{1}{2}})(s-s_a) + Yr((1+C^2(r')^2)^{\frac{1}{2}})(s^4-s_a^4). \end{cases} \tag{16}$$

For $z \in (b,c)$:

$$\begin{cases} 2B(c_3z+c_4)^3c_3(1+(Cc_3^2)^2 + AC(c_3z+c_4)^2c_3(1+(Cc_3)^2)^{\frac{3}{2}} \\ -C(1+(Cc_3)^2)(2c_3^2)\eta = 0, \\ C(1+C^2(c_3)^2)^{-1}\left(\dfrac{c_3^2}{c_3(c_3z+c_4)}\right) \\ = f_o(\eta^{-1}) + B((c_3z+c_4)^2-1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_0^Z \frac{1}{c_3}(1+C^2(c_3)^2)^{-\frac{1}{2}}dz, \\ s' = D(c_3z+c_4)((1+C^2(c_3)^2)^{\frac{1}{2}})(s-s_a) + Y(c_3z+c_4)((1+C^2(c_3)^2)^{\frac{1}{2}})(s^4-s_a^4). \end{cases} \tag{17}$$

whilst for $z \in [c,d]$ we have (since $r'' \leq 0$):

$$\begin{cases} 2Br^3r'(1+(Cr'^2)^2 + ACr^2r'(1+(Cr')^2)^{\frac{3}{2}} \\ -C^3r''(2r^2r''+r(r')^2)\eta - C(1+(Cr')^2)(2(r')^2+rr'')\eta = 0, \\ C(1+C^2(r')^2)^{-1}\left(\frac{(r')^2+2r''r}{r'r}\right) \\ = f_o(\eta^{-1}) + B(r^2-1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1})\int_0^Z \frac{1}{r'}(1+C^2(r')^2)^{-\frac{1}{2}}dz, \\ s' = Dr((1+C^2(r')^2)^{\frac{1}{2}})(s-s_a) + Yr((1+C^2(r')^2)^{\frac{1}{2}})(s^4-s_a^4). \end{cases} \tag{18}$$

all with the following parameters, material function, and scaling:

$$\begin{aligned} &A = \frac{\rho g R_o^2}{\eta_o U_o}, B = \frac{\pi \Delta p R_o^3}{\eta_o Q}, C = \frac{R_o}{Z_1}, f_o = \frac{F_o R_o}{\eta_o Q}, \beta = \frac{E}{RT_o}, \\ &D = \frac{UR_o 2\pi Z_1}{Q\rho C_v}, Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda \epsilon}{Q\rho C_v}, \\ &\eta_b = \eta_o e^{\beta(\frac{1}{s(z)}-1)}, \eta = \frac{\eta_b}{\eta_o}, z = \frac{Z}{Z_1}, r = \frac{R}{R_o}, u = \frac{U}{U_o}, w = \frac{E}{E_o} \end{aligned} \tag{19}$$

And with the following initial conditions: $r(0) = s(0) = 1$.

To get a single equation in r departing from the second equations above, one just needs to isolate Crr'' on each one of them, use the result on the first equations, and multiply the result by $\frac{2}{r'}$. This leads us to the following results:

$$2C^2r''r^2(f_o + B(r^2-1)) - r(f_o + B(3r^2-1))(1+C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} = 0. \text{ for } z \in [a,b) \tag{20}$$

and

$$-2C^2r''r^2(-f_o+B(r^2-1))-r(-f_o+B(3r^2-1))(1+C^2(r')^2)-5Cr'e^{\beta(\frac{1}{s(z)}-1)}=0.forz\in(c,d] \quad (21)$$

and we can, for perturbation purposes, consider that our ϵ is $e^{-\beta(\frac{1}{s(z)}-1)}$ once the temperature will decrease to close to zero, very close. Here, one must remember that ϵ is the standard denomination in the literature containing perturbation theory (see, for example, [11]) for the target of all the perturbational work.
What comes into play now is the bubble profile. We notice that the bubble profile has to be determined in different ways. There are two almost straight lines easily identifiable from the simulations - they are located at the beginning and at the end of the curve - respectively, that is, both towards $z = 0$ and towards the end of the profile. On the other hand, there is a region of quick change in the radius, this more suitable to the Poincare expansion than to the asymptotic approach. And, right after the first region of quick change in the profile, comes a stable region again, again almost a straight line. After this almost straight line we have another region of quick change and the curve gets finalized by another sort of straight line again. We now wish to nominate these different stages of the curve by splitting it into intervals. Each interval will be named C_x, x being a natural number from 1 to 5. We will then have:

- A straight line segment, which we call C_1, with function that expresses it being called S_1 - for $z \in [0, a)$;
- A region of quick change, which we call C_2, with a function that expresses it being called S_2 - for $z \in (a, b)$;
- Another region of stability, again an almost straight line, let's call it C_3, with function S_3 to express it - for $z \in (b, c)$;
- Another region of quick change, we call it C_4 and its curve will be named S_4 - for $z \in (c, d)$;
- Another region of stability, C_5, with function S_5 to express it - for $z \in (d, 1]$.

This way, our solution should look more or less like:

$$\begin{aligned} r(z) &= Heaviside(a-z)S_1 + Heaviside(b-z)(Heaviside(z-a))S_2 \\ &+Heaviside(c-z)(Heaviside(z-b))S_3 + Heaviside(d-z)(Heaviside(z-c))S_4 \\ &+Heaviside(1-z)(Heaviside(z-d))S_5 \end{aligned} \quad (23)$$

where $0, a, b, c, d, 1$ are the delimiters of each interval of interest where the curves lie.

In order to work out an approximate precise solution curve expression we must perform computer simulations and, through these computer simulations, we found out that (here, one must remember that C, A, B, D, Y are constants that have been previously defined):

- The value for C that gives the best curve fit was 0.16 (for all 30 different simulations performed);
- The value for A that gives the best curve fit was 0.009332488386 (for the majority of the simulations);
- The value for B that gives the best curve fit was 0.1871228447 for a single simulation where we think the best fit was achieved;
- The value for D and Y that gives the best curve fit was 0.007945372818 and 0.05.

And with the above values we get (approximately) that $a = 0.2$, $b = 0.57$, $c = 0.97$. We are sure that $r(0) = r(a) = 1$ and $dr(0) = dr(b) = 0$ since C_1 is a straight line. We emphasize that the plots we were aiming were those where the end of the curve is actually flat, the beginning is flat, to give us slope zero, so that we actually get 4 pieces, as exposed above. We performed simulations around the values used in the literature researched.

10 The first, third, and fifth pieces of the solution model

We have named the first piece of the solution model, $S_1 - z \in [0, a)$, the third piece of the solution model, $S_3 - z \in (b, c)$, and the fifth piece of the solution model, $S_5 - z \in (d, 1]$. They end up all being equivalent when it comes to the model. Therefore, it suffices reasoning over one of them to obtain the other.

For the first segment of our profile, the curve goes almost as in a straight line. Our equation is

$$\begin{cases} f_o + B(k^2-1) + \frac{A}{C}\int_0^Z (\lim_{r'\to 0}\frac{1}{r'})dz = 0, \\ s' = k[D(s-s_a) + Y(s^4 - s_a^4)]. \end{cases} \tag{24}$$

If we had not used the fact that $r' = r'' = 0$ then we would end up thinking of applying the method of the matched asymptotic expansions for C_1. In doing so, we get that our outer solution is $r^o(z) = 1$, and this is the way the profile behaves away from the boundary layer. From the exact solution we see that when $z = O(\epsilon)$, $r' = O(1)$ and $r'' = O(1)$ as well. In order to examine the "layer" region adjacent to $z = 0$, we "blow it up" by defining a local or BL variable θ defined by: $\theta = \frac{z}{\epsilon^\lambda}$ aiming to find out the right 'stretching' transformation. The choice $R(\theta,\epsilon) = r(\epsilon^{-\lambda}z,\epsilon)$ and our new equation looks like

$$2C^2\frac{d^2r}{\epsilon^{2\lambda}d\theta^2}r^2(f_o+B(r^2-1))\epsilon - r(f_o+B(3r^2-1))(1+C^2(\frac{dr}{\epsilon^\lambda d\theta})^2)\epsilon - 5C\frac{dr}{\epsilon^\lambda d\theta} = 0 \tag{25}$$

that is

$$2C^2\frac{d^2r}{d\theta^2}r^2(f_o+B(r^2-1))\epsilon^{1-\lambda} - r(f_o+B(3r^2-1))(\epsilon^{2\lambda-1}+C^2\left(\frac{dr}{d\theta}\right)^2)\epsilon^{1-\lambda} - 5C\frac{dr}{d\theta} = 0 \tag{26}$$

For $\lambda < 1, \epsilon \to 0$ and $\lambda = 1, \epsilon \to 0$ we get $R^i = K$, $K \in \Re$, and i stands for inner solution in the perturbational work. For $\lambda > 1, \epsilon \to 0$ we have to change our equation into

$$2C^2\frac{d^2r}{d\theta^2}r^2(f_o+B(r^2-1)) - r(f_o+B(3r^2-1))(\epsilon^{2\lambda-1}+C^2\left(\frac{dr}{d\theta}\right)^2) - 5C\epsilon^{\lambda-1}\frac{dr}{d\theta} = 0 \tag{27}$$

Remember, here, that R^i is standard notation for perturbation analysis (see [11], for instance), inner solution. This equation does not look very 'friendly', but may be easily compared to the result we get from the equation containing $r' = r'' = 0$. The latter is

$$\begin{cases} f_o + B(k^2-1) + \frac{A}{C}\int_0^Z (\lim_{r'\to 0}\frac{1}{r'})dz = 0, \\ s' = k[D(s-s_a) + Y(s^4 - s_a^4)]. \end{cases} \tag{28}$$

Here, one may assume that $\lim_{r'\to 0}\frac{1}{r'}$ is actually ∞ but our infinity is confined to the space where the bubble profile lives. With this, the integral might actually be workable. Once the value of the integral is found, we also determine k. But, for our solution purposes, it suffices stating that $r(z) = c_1; c_5$ in C_1 and C_5.

11 The second and the fourth pieces of the solution model

We have named the second piece of the solution model, $S_2 - z \in (a,b)$. S_2 is the expression of C_2, a region of quick change. Therefore, the best way to deal with it is using a Poincare series. We also have to work with the equation for that sector (let's consider C_2 goes from a to point b) of the profile:

$$2C_2r''r^2\epsilon(f_o+B(r^2-1)) - r\epsilon(f_o+B(3r^2-1))(1+C^2(r')^2) - 5Cr' = 0 \tag{29}$$

for $z \in (a,b)$.

By making use of a Poincare type expansion on r, we get that, for $z \in (a,b)$, our S_2 will be (taking k_1 to be a constant, yet to be fully determined):

- Since the only term without ϵ is $-5Cr'$, and changing the scaling of z to division by a does not change our equation in r, we get that $r_o'(z) = r_o''(z) = 0$ and $r_o(z) = 1$.
- From the terms in ϵ, we get that $r_1'(z) = \frac{-f_o-2B}{5C}$, what therefore implies that $r_1(z) = \frac{-f_o-2B}{5C}z + k_1$.

- From the terms in ϵ^2, we get that $r_2'(z) = \frac{-f_o-2B}{25C^2}z(-f_o-8B) + \frac{k_1}{5C}(-f_o-8B) - \frac{3BC}{5}\left(\frac{-f_o-2B}{5C}\right)^2$, what therefore implies that

$$r_2(z) = \frac{f_o+2B}{50C^2}z^2(f_o+8B) - \frac{k_1 z}{5C}(f_o+8B) - \frac{3B(f_o+2B)^2}{125C}z + k_2 \tag{30}$$

k_2 stands for a constant yet to be made precise.

With this, our S_2 can be better approximated by the following function:

$$S_2(z,\epsilon) = 1 + \left[\frac{-f_o-2B}{5C}z + k_1\right]\epsilon + \left[\frac{f_o+2B}{50C^2}z^2(f_o+8B) - \frac{k_1 z}{5C}(f_o+8B) - \frac{3B(f_o+2B)^2}{125C}z + k_2\right]\epsilon^2 \tag{31}$$

And the same sort of reasoning, the same solution, applies to C_4, that is, S_4.

12 Analytical solution

We here make use of the Heaviside function a few times in the final composition of our solution aiming considering solely the part of the curve each piece of the solution model actually refers to:

$$\begin{aligned} r(z) \;=\; & Heaviside(a-z)c_1 + Heaviside(b-z)(Heaviside(z-a))[1 + \left[\frac{-f_o-2B}{5C}z + k_1\right]\epsilon \\ & + \left[\frac{f_o+2B}{50C^2}z^2(f_o+8B) - \frac{k_1 z}{5C}(f_o+8B) - \frac{3B(f_o+2B)^2}{125C}z + k_2\right]\epsilon^2 \\ & + Heaviside(c-z)(Heaviside(z-b))[c_3 z + c_4] + Heaviside(d-z)(Heaviside(z-c)) \\ & [1 + [\frac{-f_o-2B}{5C}z + \left[\frac{f_o+2B}{50C^2}z^2(f_o+8B) - \frac{k_1 z}{5C}(f_o+8B)\right. \\ & \left. + - \frac{3B(f_o+2B)^2}{125C}z + k_2\right]\epsilon^2]] + Heaviside(1-z)(Heaviside(z-d))c_5 \end{aligned} \tag{34}$$

13 Another resolution to the problem

We approach the problem in a way similar to the way Tam approached it: we start with the removal of gravity effects in our equations, that is, we make $A = 0$. As a result of the negligence of the gravity term and basic operations with our system of equations of the simplified model presented previously in this paper, we have obtained:

$$2C^2 r'' r^2(f_o - B(r^2-1)) + r(f_o - B(3r^2-1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} = 0 \tag{35}$$

This small equation generates several possibilities. In trying to make its solution approachable via inequalities, inspired on the works of the so respectable journals in inequalities (JIPAM, Inequalities and Applications, etc.), by means of continuity theorems from analysis, we offer a few introductory reasonings. A reasonable approach to the problem, pursuing an analytical solution via inequalities, would be considering that the minimum the function $r(z)$ reaches is $r(0) = 1$ and the maximum is $r(1) = A$. Therefore, one could say that

$$\begin{aligned} & 2C^2 r'' f_o + (f_o - 2B)(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \\ < \;& -2C^2 r'' r^2(-f_o + B(r^2-1)) - r(-f_o + B(3r^2-1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \\ < \;& -2C^2 r'' A^2(-f_o + B(A^2-1)) - A(-f_o + B(3A^2-1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \end{aligned} \tag{36}$$

We also know that $s(0) = 1$ is the maximum temperature reached in the whole process, where the temperature varies between 0 and 1. Therefore, we can also say that:

$$\begin{aligned} & 2C^2r''f_o + (f_o - 2B)(1 + C^2(r')^2) - 5Cr'e^{\beta} \\ < \quad & -2C^2r''r^2(-f_o + B(r^2-1)) - r(-f_o + B(3r^2-1))(1 + C^2(r')^2) - 5Cr'e^{(\frac{1}{s(z)}-1)} \\ < \quad & -2C^2r''A^2(-f_o + B(A^2-1)) - A(-f_o + B(3A^2-1))(1 + C^2(r')^2) - 5Cr'e^{(\frac{1}{s(z)}-1)} \end{aligned} \quad (37)$$

If we are then able to find a solution for the equations

$$r'' = \frac{5Cr'e^{\beta} - (f_o - 2B)(1 + C^2(r')^2)}{2C^2 f_o} \quad (38)$$

and

$$r'' = \frac{5Cr'e^{\beta}(\frac{1}{s(z)} - 1) + A(-f_o + B(3A^2-1))(1 + C^2(r')^2)}{-2C^2A^2(-f_o + B(A^2-1))} \quad (39)$$

we then are going to be able to say that our solution is between those two.

14 Conclusions

We presented an initial analytical investigation into the bubble radius profile of a nonisothermal film blowing process, which was never made before. The present work must have a follow-up with trials to obtain analytical solutions departing from the solutions we presented in this paper, where we took away the gravity term and left part of the solution as a question mark. Because of the value of A, our solution might be very close to reality. So far, however, we have not written about our empirical notes, obtained from [6], in order to check on proximity to real life situations. Future work must bring this sort of results along with analytical trials on getting more of our solutions, even, possibly, revealing the value of our constants.

With this work, our major intention was presenting alternative ways to approach the solution to the radius function determination and make some good progress towards them. We hope the reader understands that we suggest an analytical path via inequalities that might even allow us to get theorems referring specifically to that bubble problem. As for our reserve of market, we must state that in our next approach we are going to worry about how relevant to the industry our results might be once, so far, our comparisons with the actual life models have been very weak, even though, quite well succeeded.

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