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Optimum design of suspension system of three—wheeled motor vehicles*

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Abstract. Three - wheeled motorized vehicles play an important part in city transport of developing countries. In the present work, the results of an optimum design of suspension system using 9 degrees-of-freedom (DOF) analytical model for coupled motion of commercial three - wheeled motor vehicles of Bajaj rear engine (RE) and Vikram front engine (FE) vehicle over road of various degree of measured roughness are presented. Using ride comfort criteria, the significant design variables affecting system behaviour namely spring stiffness and viscous damping values of the front and rear suspension, wheelbase and track width were optimized using random search optimization technique (RST) proposed by Mohan and Deep[7]. The resulting ride behaviour has been compared with International Standard Organization (ISO) 2631 values[10]. The obtained Pareto – optimal solutions show favorably good results as compared with original and parametric analysis results. In this optimization method, the sensitivity of the eight design variables towards the minimization of root mean square acceleration spectral density using two methods is ascertained and is reported.

Keywords: three–wheeled vehicles, random search technique, root mean square acceleration response, Pareto–optimal solutions, Bajaj rear engine vehicle and Vikram front engine vehicle

1 Introduction

The optimization process of vehicle suspension system involves: (a) Modeling and analyzing the vehicle behaviour. (b) definition of the optimization objective, and the specifications for any proposed solution. (c) choosing an appropriate methodology to satisfy the design objectives and (d) using mathematical programming techniques to optimize the vehicle and suspension parameters to satisfy one or more criteria.

The primary objective of the three-wheeled vehicle suspension design is to provide sufficient vibration isolation in different directions due to road disturbances, so that the desired level of comfort for the driver and passengers is obtained. In general, vehicle suspension characteristics and other parameters (like mass, inertia and geometrical parameters) have been well known to affect the ride behaviour and are of interest to the vehicle dynamicist. The optimum values of parameters which minimize the discomfort of the driver and passengers are obtained. The literature review shows, very few publications on three-wheeled vehicles, which mostly deal with the stability and handling characteristics of three wheelers. After going through the available literature in the field of ride dynamics and optimization of three wheeled motor vehicles, it has been found to the best of knowledge of the authors; there is a lack of availability of literature regarding the ride dynamics and optimization of three - wheeled vehicles. However, some published references were made relating to the ride behaviour and optimization of two and four wheeled motor vehicles. The previous work relating to

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optimization behaviour of vehicles other than three wheeled vehicles gives some idea in the evaluation of optimum ride response of the vehicles. Very few publications[14–16, 18] dealing with ride behaviour of three-wheeled vehicles are available.

Several researchers[2–9, 11–13, 17, 18] used linear and non-linear mathematical programming optimization techniques to minimize the RMS accelerations of the driver and passengers in the field of vehicle dynamics. In all these methods, sensitivity of the design variables towards minimization of acceleration spectral density is ascertained. These were the earliest techniques available for solving non – linear optimization problems for obtaining local optimum solution. In 1980's a number of techniques came up in literature for obtaining the global optimal solution of non – linear optimization problems. Lately Fuzzy approach, Neural networks approach, Simulated Annealing approach and Genetic algorithm have come up for solving the global optimal solution of non – linear optimization problems. Obtaining the global optimal solution of a nonlinear optimization problem is much more difficult than obtaining a local optimal solution. However, in certain situations of practical interest, it is necessary to obtain a global solution rather than a local optimal solution. Keeping this in view, several methods are now being proposed in the literature for obtaining a global optimal solution of a nonlinear optimization problem.

Afimiwala and Mayne[2] considered the optimum design of an impact absorber. The system was subjected to step change in base velocity and the objective of the design was to minimize the maximum acceleration level occurring during the transient response. Elmaraghy, Dokainish and Siddall[9] used non-linear mathematical programming techniques to minimize the maximum acceleration of the driver of a railway vehicle. Dahlberg[5] studied with random input a 5-DOF vehicle model by using the technique of input – output relations for spectral densities. A computer program has also been developed for optimization of two or more of the system parameters to make a vehicle response (or a weighted sum of responses) a minimum. Nack[11] developed algorithms for minimization of displacement or acceleration (as an objective function) and formulated as a Min-Max optimization problem to determine vibrational response of structural systems. Duncan[7] used a finite element analysis to improve the ride quality of a light duty truck, using an optimization technique.

On the basis of parametric study, the following parameters, which influence the ride behaviour most, have been identified and are included in the optimization scheme.

- Stiffness and damping of front and rear suspension systems
- Wheelbase of three wheeled vehicle
- Track width of the vehicle

In the present analysis, a computational algorithm[10] for finding global optimal solutions of nonlinear optimization problems has been used and is coded in C++.

The optimization process minimizes either the vertical root mean square (RMS) acceleration or the lateral RMS acceleration. Based on the results obtained from the vertical and lateral ride and parametric analysis, lateral root mean square acceleration response (RMSAR) values for Vikram and Bajaj vehicles are always well within the International Standard Organization (ISO) limits in the entire frequency range from 0.1 to 80 Hz, whereas vertical RMSAR exceeds the ISO ride comfort limits in the mid frequency range. Therefore optimization has been performed to minimize only vertical root mean square (RMS) acceleration at the center of gravity (c.g.) of the sprung mass, resulting in slight penalty of the vehicle ride behaviour in the lateral direction. However the resulting lateral RMS acceleration values for the original and parametric analysis were found to be still well below the ISO limits. The objective of the present study is to choose the design variables of the vehicle under excitation due to road roughness, so as to obtain optimized ride behaviour in vertical direction.

2 Problem formulation

The three-wheeled vehicles for the present study have been modeled as four and five mass systems for Vikram and Bajaj vehicles respectively. From a dynamical point of view, vehicle can be regarded as a multibody system consisting of a sprung mass, steering arm, front wheel unsprung mass, and a solid rear axle comprising the rear unsprung mass in case of Vikram front – engine vehicle. The Bajaj vehicle has independent suspension system in the rear, and is therefore modeled as a five mass system. The "rigid body" assumption is valid in the frequency range of interest (0.1–10 Hz.) since the vehicle motion modes frequencies are in this range only and it has been shown in the literature that vibration modes due to structural flexibility occur much above 10 Hz.

The degrees of freedom associated with vehicle model consist of 6-DOF for the sprung mass, 2-DOF for the rear unsprung mass relative to the sprung mass, and 2-DOF for the front wheel bounce and the angular motion of the steering arm. For the sprung mass the three translational coordinates are longitudinal, lateral and bounce displacements and three rotational coordinates are the Euler angles in xyz-convention representing its yawing, rolling and pitching motions. The vehicle speed is considered to be constant which reduces the sprung mass degrees of freedom to five. A set of equations is obtained for an assumed 9-DOF mathematical model of vehicle - tyre system using Lagrangian dynamics approach.

The exact nature of the optimization problem, its objective function and restrictions, depend on the type of excitation considered. The random search technique (RST) proposed by Mohan and Deep[10] for global optimization for solving nonlinear global optimization problems has been used in the present work to optimize the design variables for obtaining minimization of vertical RMSAR in the frequency range of 0.1 to 80 Hz. Selecting a multi-objective function and allowing suspension and vehicle parameters to vary over a wide range with imposition of constraints, the optimal values of the design variables have been obtained from a passenger ride comfort viewpoint for a rough road excitation.

In order to evaluate the performance of a vehicle, a specific index such as ride quality (RMSAR) has been used. The objective function quantifies the vehicle comfort level represented by RMSAR or its square and is nonlinear in nature. Based on previous clarification, therefore only the vertical RMSAR has been considered as objective function to minimize from the ride comfort view point.

ISO 2631[1] estimates the fatigue time by computing the RMSAR weighted at the output of the filter with band pass characteristics in the range of 0.1 to 80 Hz. In accordance with the weighing curves, the weighted root mean square acceleration response is given by:

$$RMSAR = \sqrt{\int_{\omega_1}^{\omega_2} \omega^4 B\left(\omega\right) S_{q_r}(\omega) d\omega} \tag{1}$$

where $S_{q_r}(\omega)$ is the spectral density function for displacement $q_r(t)$ and $[\omega_1, \omega_2]$ is the frequency interval in which it is computed and $B(\omega)$ is weighting factor.

The ISO fatigue time is inversely proportional to RMSAR. Therefore, the objective function $f(x)$has been taken as to minimize square of RMSAR by using (1) and by taking appropriate weighing factors at different frequencies as proposed in ISO 2631[1].

$$f(X) = \int_{\omega_1}^{\omega_2} \omega^4 B\left(\omega\right) S_{q_r}(\omega) d\omega \tag{2}$$

3 Description of RST technique

The RST algorithm can be used to solve constrained nonlinear global optimization problem of the type:

$$\begin{aligned} & min\, f(X) \\ & \text{s.t.} \begin{cases} g_i(X) \geq 0, & i = 1, 2, 3, \cdots, m; \\ x_k^{(L)} \leq x_k \leq x_k^{(U)}, & \text{where } k = 1, 2, 3, \cdots, n; \\ X = (x_1, x_2, x_3, \cdots, x_n). \end{cases} \end{aligned} \tag{3}$$

The RST algorithm works iteratively in two phases and depends on evaluation of function alone. It assumes no specific properties (such as differentiability or continuity) of the functions appearing in the problem. In the first phase, the objective function is evaluated at a number of randomly sampled feasible points, while in the second phase these points are manipulated by local searches (using quadratic approximation) to yield possible candidates for the global optimum.

However, since the problem in the present work has been modeled as a nonlinear optimization problem with multi-objective function, the RST also has been modified to solve multi-objective function of the type

$$
\begin{aligned}
&min\ \{f_1(X), f_2(X), f_3(X), \cdots, f_t(X)\}\\
&\text{s.t.} \begin{cases} g_i(X) \geq 0, & i = 1, 2, 3, \cdots, m;\\ x_k^{(L)} \leq x_k \leq x_k^{(U)}, & k = 1, 2, 3, \cdots, n;\\ X = (x_1, x_2, x_3, \cdots, x_n). \end{cases}
\end{aligned}
\tag{4}
$$

where $f_1, f_2, \cdots, f_t$ are t number of objective functions that have to be minimized simultaneously.

In the present work two approaches have been used for solving the multi - objective functions. The first method of multi–objective optimization problem, in which the main objective is to minimize sum of the weighted values of vertical RMSAR in the frequency range of 0.1 to 80 Hz.

$$
\begin{aligned}
&min\, f(X) = \sum_{i=0.1}^{80} f_i(X)\\
&\text{s.t.} \begin{cases} g_i(X) \geq 0, & i = 1, 2, 3, \cdots, m;\\ x_k^{(L)} \leq x_k \leq x_k^{(U)}, & k = 1, 2, 3, \cdots, n;\\ X = (x_1, x_2, x_3, \cdots, x_n). \end{cases}
\end{aligned}
\tag{5}
$$

In the second method the objective function, representing design variables that restrain RMSAR values to be less than a maximum value, is used basically in Min. Max. approach. The minimization of RMSAR values can be formulated as a Min-Max optimization problem i.e.

$$
\begin{aligned}
&min\, [max.(f_i(X))]\\
&\text{s.t.} \begin{cases} g_i(X) \geq 0, & i = 1, 2, 3, \cdots, m;\\ x_k^{(L)} \leq x_k \leq x_k^{(U)}, & k = 1, 2, 3, \cdots, n;\\ X = (x_1, x_2, x_3, \cdots, x_n). & \\ i = 1, 2, 3, \cdots, t & \end{cases}
\end{aligned}
\tag{6}
$$

In the above two approaches, multi objective optimization problem give rise to a set of optimal solutions known as Pareto – optimal solutions, all of which are equally important as far as all objectives are concerned. By taking parametric analysis into consideration, the appropriate design vector solution is recommended out of the number of available solutions.

4 Computational steps of RST technique

The computational steps of the algorithm are summarized below. The RST algorithm[10] is used for obtaining the global optimal solution of non linear optimization problems as defined above. Instead of working with just one solution, this algorithm works for a random sample of solutions. The algorithm works in two phases. In the first phase the objective function is evaluated at a number of randomly generated feasible solutions. In the second phase, these solutions are manipulated by local searches (using quadratic approximation) to yield possible candidates for the global minimum. The iterative process stops when all the feasible solutions converge to the global minimum.

First Phase

(i) Choose a suitably large number of N (say $N = 10(n + 1)$) random feasible points and evaluate the objective function at these points. Store these points and their function values in an N by $(n + 1)$ array A. Set ITER $= 1$, MULT $= 1$.

(ii) Out of these N points find M and L, the points with the greatest and the lowest function values $f(M)$ and $f(L)$, respectively. If $|(f(M) - f(L))/f(M)| < \varepsilon$ stop with the message that L is the point of global minima, else set IFAIL $= 1$ and go to (iii).

Second Phase

(iii) From the array A choose three points $B_1 = L$, B_2 and B_3 randomly and determine the next trial point P, as the point of minima of the quadratic curve passing through B_1, B_2 and B_3 (the point where the gradient of the quadratic curve through these three points is zero). Explicitly, this point P is given by:

$$P = 0.5 \times \left\{ \frac{(B_2^2 - B_3^2)f(B_1) + (B_3^2 - B_1^2)f(B_2) + (B_1^2 - B_2^2)f(B_3)}{(B_2 - B_3)f(B_1) + (B_3 - B_1)f(B_2) + (B_1 - B_2)f(B_3)} \right\}$$

If the co-ordinates of $P(p_1, p_2, p_3, \cdots, p_n)$ lie within the specified feasible range $a_j \leq p_j \leq b_j, j = 1, 2, 3, \cdots, n$ then go to (iv) otherwise set $p_j = b_j$ for those j for which $p_j > b_j$ and $p_j = a_j$ for those j for which $p_j < a_j$ and go to (iv).

(iv) Check if P satisfies the constraints $g_i(X) \geq 0$. If P is feasible then go to (v), otherwise go to (vi).

(v) Find $f(P)$. If $f(P) > f(M)$ go to (vi), otherwise replace the current M by P in array A and set ITER $=$ ITER $+ 1$. If ITER $>$ ITL(where ITL is some pre assigned positive integer specifying the upper bound on number of permissible iterations ITER) go to $(viii)$, otherwise go to (ii).

(vi) Set IFAIL $=$ IFAIL $+ 1$. If IFAIL $>$ LAST go to (vii), else go to (iii).

(vii) If MULT $>$ MLAST go to $(viii)$, otherwise set MULT $=$ MLAST $+ 1$ and replace the worst $(N - n)$ points of array A by new randomly generated feasible points, and go to (ii).

$(viii)$ Stop with the indication that the current L is the best point which could be found by the algorithm.

5 Results and discussions

Considering the objective function given in (2), vertical RMSAR on institute road for the Bajaj and Vikram vehicles has been minimized. The vertical and lateral ride behaviour, for the purpose of this study, has been evaluated at 45km./h., which is normally the vehicle speed in city travel. Eight different vehicle and suspension parameters have been considered in the optimization process. In the random search optimization technique the variation of $\pm 60\%$ of the original values has been considered as the upper and lower bounds for all the suspension parameters (stiffness and damping) as one of the constraints and necessary input for RST. Further in the optimization process the stiffness and damping characteristics of left and right rear suspensions are taken same for both the vehicles. The wheelbase and track width are two other vehicle parameters affecting ride behaviour and therefore included in the optimization function. Since these parameters affect vehicle handling and longitudinal dynamics as well to a large extent and are important vehicle design considerations in terms of their maneuverability in congested traffic conditions, therefore, the variation in wheelbase and track width has been limited to $\pm 20\%$ with the vehicle c.g. remaining in the longitudinal plane. For an assumed multi-criterion function, three solutions (representing same set of system parameters) each for the two optimization methods using (5) and (6) for both vehicles have been obtained. The corresponding sets representing optimal values of system parameters are given in Tab. 1 for Vikram and in Tab. 2 for Bajaj vehicle. The vertical and lateral RMS acceleration responses at the c.g. of the sprung mass for Vikram (Fig. 1 $\sim$ Fig 4) and Bajaj (Fig.5 $\sim$ Fig 8) vehicles have been shown. The results of Fig. 1 and Fig. 2, indicate that considerable improvement in vertical ride behaviour for Vikram vehicle is obtained for all the six sets of vehicle parameters. However out of the six solutions, the ride behaviour (vertical RMSAR) for sum. min. (set1) and sum. min. (set 3) are giving slightly higher RMSAR values as compared with RMSAR values for the remaining four sets of vehicle parameters. In addition to this, the values of RMSAR for all the sets of optimized parameters are better in the entire frequency range with a slight penalty between 0.6 to 1.25 Hz. Fig. 3 and Fig. 4 shows the lateral RMS acceleration response for the original set of vehicle parameters and for the six sets of optimized parameters for Vikram vehicle. The results indicate that lateral RMS acceleration values are better for the proposed optimized sets in the entire frequency range and all are well within the ISO limits.

The vertical and lateral RMS acceleration response curves, Fig. 5 $\sim$ Fig. 8 for Bajaj vehicle, indicate that all the six Pareto-optimal solutions lead to considerable improvement in lateral and vertical ride behaviour in the entire frequency range. However with respect to vertical and lateral RMS acceleration response, Fig. Fig. 7 and Fig. 8 indicates that peaks of bounce and lateral modes are respectively shifting towards 2.1 and 2.0 Hz

Table 1. The original and optimal parameters of three-wheeled Vikram vehicle by using-min.-max. Approach and Weighted-Sum Approach

S. No.	Notation	Original	min-max.approach			Weighted-Sum Approach		
			Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)
1	k_1 (N/m)	86700.0	61760.6	59833.3	53142.6	54448.8	49229.2	56880.9
2	k_2 (N/m)	79410.0	32183.4	32386.8	31764.0	31864.5	31764.0	31885.0
3	k_3 (N/m)	80000.0	32183.4	32386.8	31764.0	31864.5	31764.0	31885.0
4	c_1 (Ns/m)	1496.5	1576.7	1080.8	1377.4	837.8	818.6	855.1
5	c_2 (Ns/m)	1213.0	1385.6	1820.2	1819.2	1302.1	949.9	1391.3
6	c_3 (Ns/m)	1213.0	1385.6	1820.2	1819.2	1302.1	949.9	1391.3
7	tw (m)	1.168	1.371	1.31	1.345	1.402	1.398	1.399
8	Wb (m)	1.864	1.709	1.428	1.437	1.559	1.759	1.528

Table 2. The original and optimal parameters of three - wheeled Bajaj vehicle by using-min.-max. Approach and Weighted-Sum Approach

S. No.	Notation	Original	min-max.approach			Weighted-Sum Approach		
			Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)	Optimal (set 1)
1	k_1 (N/m)	32736.0	14525.4	15039.3	18954.1	13330.6	13224.1	13967.9
2	k_2 (N/m)	50400.0	20078.0	19920.0	20440.9	19920.0	20077.7	20348.8
3	k_3 (N/m)	49800.0	20078.0	19920.0	20440.9	19920.0	20077.7	20348.8
4	c_1 (Ns/m)	3500.0	1300.0	1420.4	1321.7	1306.3	1309.5	1314.6
5	c_2 (Ns/m)	2207.5	1305.3	1329.6	1294.0	1302.9	1298.8	1294.0
6	c_3 (Ns/m)	2207.5	1305.3	1329.6	1294.0	1302.9	1298.8	1294.0
7	tw (m)	1.15	1.364	1.378	1.37	1.372	1.365	1.374
8	Wb (m)	2.0	2.058	2.009	2.058	2.186	2.205	2.204

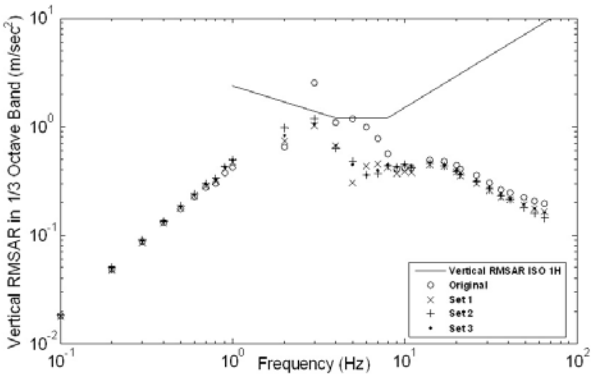

Fig. 1. Vertical RMSAR of Vikam Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (min-max approach)

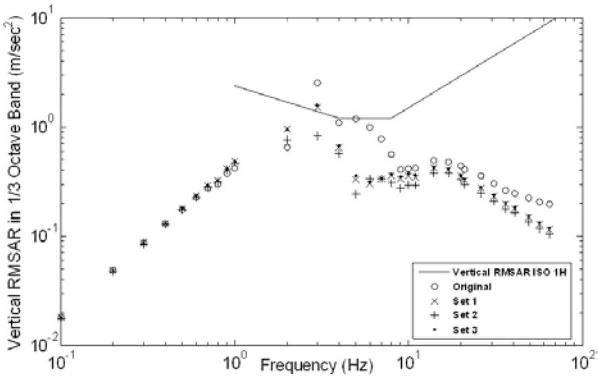

Fig. 2. Vertical RMSAR of Vikam Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (weighted sum approach)

instead of 2.8 and 2.9 Hz. Therefore important observation is that the natural frequencies in vertical bounce and lateral modes for Bajaj vehicle are lower for the recommended optimized design vector. This is further confirmed by the eigenvalue analysis, the results of which are given in Tab. 3. The lateral acceleration response for the original vehicle and for the vehicle with optimized parameters is well within the ISO limits in the entire frequency range. Six multi-criterion (Pareto – optimal) solutions for multi-criterion objective function

Table 3. Comparison of natural frequencies for optimized design vector and eigenvalue analysis in vertical and lateral modes of Bajaj vehicle

S.No.	Optimum Solution	Eigenvalue analysis	Mode Description
1	2.1 Hz	1.92 Hz	Sprung mass bounce mode
2	2.0 Hz	1.89 Hz	Sprung mass Lateral mode

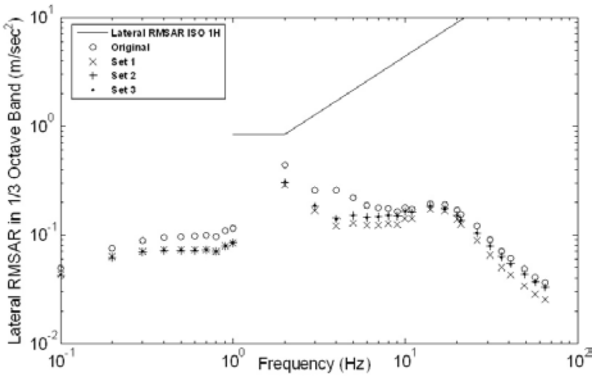

Fig. 3. Lateral RMSAR of Vikam Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (min-max approach)

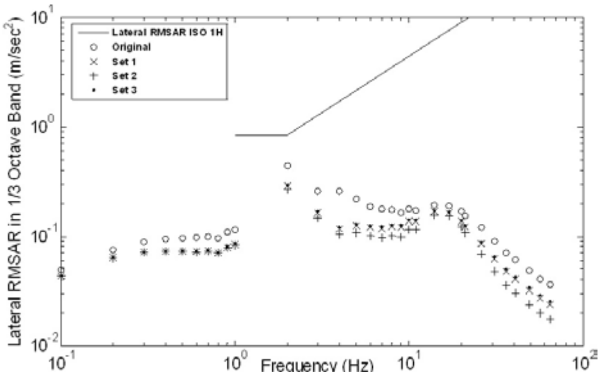

Fig. 4. Lateral RMSAR of Vikam Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (weighted sum approach)

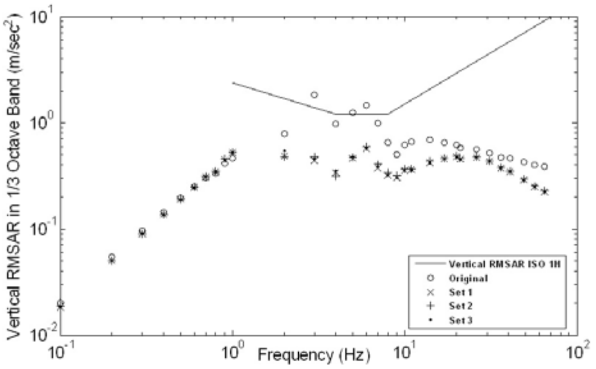

Fig. 5. Vertical RMSAR of Bajaj Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (min-max approach)

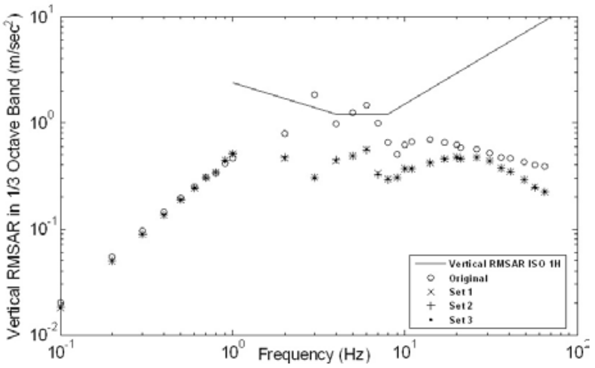

Fig. 6. Vertical RMSAR of Bajaj Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (weighted sum approach)

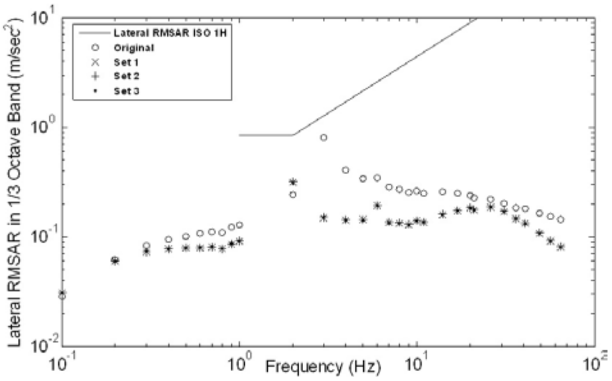

Fig. 7. Lateral RMSAR of Bajaj Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (min-max approach)

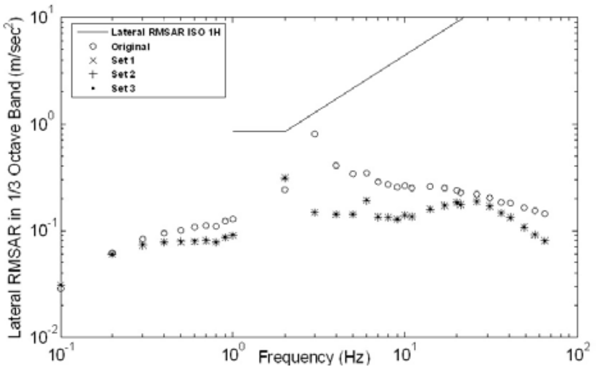

Fig. 8. Lateral RMSAR of Bajaj Vehicle Sprung Mass Center for Various Sets of Optimized Vehicle Parameters (weighted sum approach)

have been provided for optimized ride behaviour of the two commercial vehicles. The possible solutions provide considerable flexibility to the vehicle manufacturer selecting a particular set of vehicle parameters after considering other issues such as handling, longitudinal dynamics, maneuverability and manufacturing.

Considering improvement in vertical and lateral RMS acceleration response, min. max. (set3) of vehicle parameters (Tab. 1 and Tab. 2) is recommended for both vehicles. The vertical response behaviour for Vikram and Bajaj vehicles is shown in Fig. 9 and Fig. 10 respectively for original and recommended set of vehicle parameters. Fig. 9 indicates that vertical RMS acceleration of Vikram vehicle is vastly improved in the entire frequency range and is now with in ISO limits. The percentage improvements in bounce and pitch mode

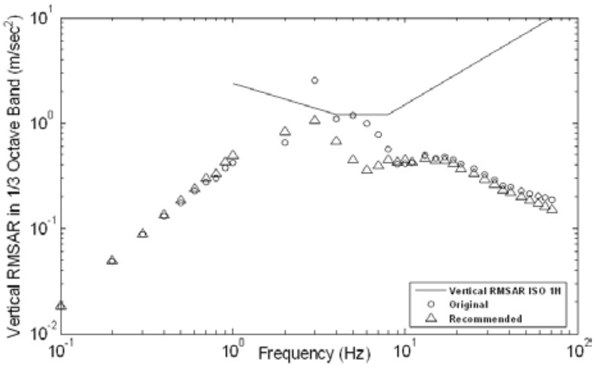

Fig. 9. Vertical RMSAR of Vikam Vehicle Sprung Mass Center for Original and Recommended set of Optimized Vehicle Parameters

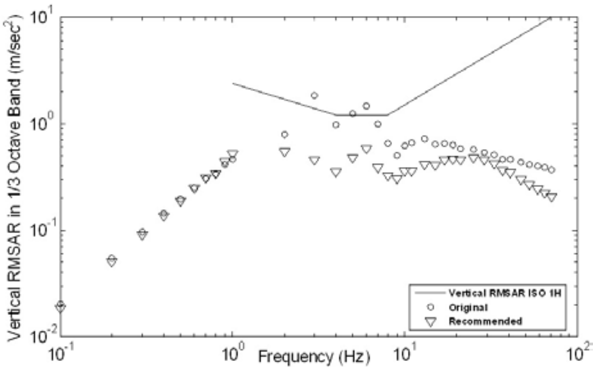

Fig. 10. Vertical RMSAR of Bajaj Vehicle Sprung Mass Center for Original and Recommended set of Optimized Vehicle Parameters

values is 60 and 65 percent respectively for Vikram. Fig. 10 shows the vertical response behaviour of Bajaj vehicle with recommended set of vehicle parameters along with the response behaviour for original set of parameters. It is found that the vertical response behaviour of Bajaj vehicle with the recommended set of vehicle parameters is drastically reduced in the entire frequency range and is well within ISO limits. The maximum vertical RMS acceleration is reduced by 70% and 57% in bounce and pitch modes respectively.

From the parametric analysis, the recommendations for Vikram vehicle are: 30% decrease in suspension damping and 40% decrease in suspension stiffness values. For Bajaj vehicle, the design recommendations include 30% decrease in suspension stiffness and 30% decrease in suspension damping. Track width has to be increased by 15% over the existing values to obtain better ride behaviour for both vehicles. For Bajaj vehicle wheelbase may be decreased by 20% and for Vikram vehicle wheelbase has to be decreased by 15%. Though increases in sprung mass and moment of inertia values are better, they are practically not desirable from the manufacturer's point of view. Finally caster trail and rake angle have no effect on vertical and lateral ride behaviour.

6 Summary and conclusions

Random search optimization technique has been suggested for minimizing the RMSAR of the vehicle suspension system, by taking the desired boundary values of RMSAR and by taking appropriate weighting factors into consideration for Vikram and Bajaj three-wheeled vehicles and suspension system.

Based on the parametric analysis, design changes are proposed to achieve better vertical and lateral response behaviour. However, suspension and vehicle parameters are optimized using RST optimization method and obtained better response characteristics than the parametric analysis.

Pareto optimal solutions are obtained for Vikram and Bajaj vehicle using multi objective function by using minimization method and min. max. Optimization solution techniques. From the manufacturer view point lots of options are there for choosing best optimal solution from the number of available solutions.

The results of the analysis presented in this paper can be used at the design stage of the three-wheeled vehicle and suspension system, for approaching an optimized system from the start.

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