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An equivalent inductance value for a time varying inductor

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Abstract. As the current increases, for a typical magnetization curve of an iron core coil, the magnetic flux density will increase. A point is reached, however, where further increases in current yield smaller and smaller increases in flux density. This is called the saturation point, and is characterized by a dramatic change in the slope of the current-flux density curve. The slope of this curve is proportional to the inductance of the coil. Geomagnetically induced currents that flow on the earth surface due to Geomagnetic Disturbance are considered one of the reasons for changing inductance values in power transformers. They are typically 0.001 to 0.1 Hz and could reach peak values as high as $200A$, they can enter transformer windings and bias the transformer cores to cause half cycle saturation. GIC is slowly varying compared with the ac frequency and are often referred to as quasi-dc. The inductance of the flux path outside the core during saturation is considerably less than the large inductance of the unsaturated core. Thus the switching of the transformer in and out of saturation each ac cycle is accompanied by a corresponding switching between two quite different inductance values. In this paper an expression for the equivalent inductance is derived which makes it possible to determine the transformer inductance presented to GIC as a function of transformer saturation.

Keywords: variable inductor, induced currents, transformer, switching, saturation, magnetic effect

1 Introduction

Geomagnetically induced currents (GIC) are driven by the electric fields produced by the magnetic field variations that occur during a geomagnetic disturbance[2, 4, 6–9]. Because of their low frequency compared to the ac frequency, the geomagnetically induced currents appear to a transformer as a slowly-varying dc current. GIC flowing through the transformer winding produces extra magnetization which, during the half-cycles when the ac magnetization is in the same direction, can saturate the core of the transformer[1, 3]. This results in a very spiky ac waveform with increased harmonic levels that can cause malfunction of relays and other equipment on the system and lead to problems ranging from trip-outs of individual lines to collapse of the whole system. The dc shifts the normal operating point of the transformer so that, during that part of the ac cycle when the ac and stray current magnetization combine, the transformer core becomes saturated. Any asymmetry in the saturation (detected by measuring the 2nd harmonic of the applied signal) in instruments that are deliberately driven into saturation by an applied ac field; is proportional to the magnetic field enveloping the instrument. There are well established expressions[5] for the equivalent inductance of a group of inductive elements, such as transformer windings, whose values do not change with time. However, little work has been done to determine the equivalent inductance of time-varying inductive elements. This paper examines the effect of an inductance that switches repeatedly between two values. Also an expression is derived for the equivalent inductance seen by currents that are varying slowly compared with the rate of switching of the inductance values.

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2 Equivalent inductance derivation

For an R-L circuit shown in Fig. 1, let the applied voltage source V, where the source consists of both an ac component and a slowly varying dc component and consider that the switch is closed at time t_0 and the inductor has a value of L_1 (saturated value) from time $t = t_0$ to $t = t_1$ and value L_2 (unsaturated) from $t = t_1$ to $t = t_2$.

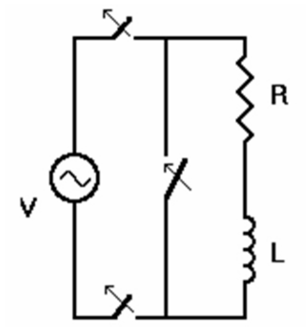

Fig. 1. An r-l circuit with the excitation voltage removed, as the direction of switches indicate, at $t = t_0$

The current will decay in two stages, at $t = t_1$ the current through the inductor is given by:

$$I_1 = I_0 \exp\left(-\frac{(t_1 - t_0)}{T_1}\right) \tag{1}$$

where: $T_1 = \frac{L_1}{R}$.

Similarly, at $t = t_2$ the current is

$$I_2 = I_1 \exp\left(-\frac{(t_2 - t_1)}{T_2}\right) \tag{2}$$

where: $T_2 = \frac{L_2}{R}$.

Substituting for I_1 gives

$$I_2 = I_0 \exp\left(-\frac{(t_1 - t_0)}{T_1}\right) \exp\left(-\frac{(t_2 - t_1)}{T_2}\right). \tag{3}$$

Equation (3) can be rearranged to give:

$$I_2 = I_0 \exp\left(-\left(\frac{(t_1 - t_0)}{T_1} + \frac{(t_2 - t_1)}{T_2}\right)\right). \tag{4}$$

A general form of equation (4) can be rewritten to take into account subsequent change in inductance value as:

$$I_n = I_0 \exp\left(-\left(\frac{(t_1 - t_0) + \cdots + (t_{n-1} - t_{n-2})}{T_1} + \frac{(t_2 - t_1) + \cdots + (t_n - t_{n-1})}{T_2}\right)\right). \tag{5}$$

Back to equation (3) which can be rewritten as

$$I_2 = I_0 \exp\left[-(t_2 - t_0)\left(\frac{(t_1 - t_0)}{(t_2 - t_0)T_1} + \frac{(t_2 - t_1)}{(t_2 - t_0)T_2}\right)\right]. \tag{6}$$

This is simplified to:

$$I_2 = I_0 \exp\left[-t\left(\frac{\Delta}{T_1} + \frac{(1-\Delta)}{T_2}\right)\right]. \tag{7}$$

The time, $t = (t_2 - t_0)$ is the total time and $\Delta = \frac{(t_1-t_0)}{(t_2-t_0)}$ and $(1-\Delta) = \frac{(t_2-t_1)}{(t_2-t_0)}$ are the proportion of this time for which the inductor has values L_1 and L_2, respectively.

Equation (7) can be rewritten in terms of an equivalent time constant as

$$I = I_0 \exp\left(-\frac{t}{T_{eq}}\right) \tag{8}$$

where

$$\frac{1}{T_{eq}} = \frac{\Delta}{T_1} + \frac{(1-\Delta)}{T_2} \tag{9}$$

This time constant can be related to an equivalent inductance

$$T_{eq} = \frac{L_{eq}}{R}. \tag{10}$$

Then substituting for $T_{eq}T_1$ and T_2 in (9) and dividing by R gives the expression for equivalent inductance

$$\frac{1}{L_{eq}} = \frac{\Delta}{L_1} + \frac{(1-\Delta)}{L_2}. \tag{11}$$

Therefore, the current in the circuit at any time t, could be determined directly from equation (8) by using the equivalent inductance equation (11) to calculate the equivalent time constant for the exponential decay. Equation (11) is rearranged after dividing both sides by L_1 as:

$$\frac{L_{eq}}{L_1} = \left[\Delta + \frac{(1-\Delta)}{L_2/L_1}\right]^{-1}. \tag{12}$$

It can be seen that, when the proportion of time in saturation is large compared to the ratio of the inductance values, the term involving the large unsaturated inductance becomes insignificant and the equivalent inductance is dependent only on the saturated inductance and the degree of saturation. That is,

$$\frac{L_{eq}}{L_1} = \frac{1}{\Delta}. \tag{13}$$

In a large power transformer the unsaturated inductance can be one thousand times the saturated inductance and so this condition is satisfied for most values of Δ. Thus, beyond very mild saturation, the equivalent inductance of a saturated power transformer is inversely related to the proportion of time in saturation.

3 Conclusion

It is shown that the equivalent inductance seen by slowly varying currents flowing through a time-varying inductance is given by

$$\frac{1}{L_{eq}} = \frac{\Delta}{L_1} + \frac{(1-\Delta)}{L_2}$$

where L_1 and L_2 are the inductance values and Δ is the proportion of time that the inductance has valueL_1. The above expressions were derived for current components varying slowly with respect to the rate of switching between inductance values.

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