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Investigation on the vibration characteristics of a sandwich beam with smart composites — MRF

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Abstract. This paper establishes the vibration model of the smart composites beam featuring magnetorheological fluid (MRF). The vibration analysis is finished under different magnetic field strength. The results show that the natural frequencies and loss factors of the MRF beam are increased with increasing applied magnetic field strength. The structural vibration responses are reduced significantly. The vibration of sandwich beam with MRF gets effective suppression for applied magnetic field.

Keywords: smart composites, magnetorheological fluid, sandwich beam, natural frequency, loss factor

1 Introduction

Magnetorheological fluids (MRF) have great potential in applications for intelligent materials and structures. MRF's physical properties such as viscosity and shear modulus can vary when subjected to different magnetic fields[3, 5]. The use of an MRF for the construction of smart components has been previously suggested and investigated[1, 3, 11].The MRF has the same properties as a viscoelastic material at small strain level[9, 14].

The problem of dissipating energy in structures so as to reduce vibration and noise, and to avoid fatigue failure, is becoming an increasingly important consideration in many areas (e.g., mechanical design, aerospace industry, civil engineering). Constrained layer damping treatment has been an effective way to suppress structural vibration, especially for the vibration control of flexible structures. Early fundamental work on the analysis of sandwich beams was conducted by Ross and his coworkers, which established the concept of using complex elastic moduli for the analysis of the laminated sandwich beam[6, 8]. The RKU (Ross, Kerwin, Ungar) model is developed for simply supported boundary conditions. Subsequently, numerous researchers developed models based upon the concepts of a thin damping layer. Mead & Markus[7] developed a transverse vibration model, resulting in a six order differential equation of motion. The MM model is developed for generalized boundary conditions.

Investigations of the MRF in the structural vibration begins just only by Yalcintas et al and Qing Sun et al[9, 13, 14]. They studied the vibration problem of a sandwich beam with an MRF core and discussed the variable properties of MRF in structural vibration control. They developed a theoretical model based on the energy method to predict the vibration characteristics, and proved the model validity through experimental investigation.

In this study, the natural frequencies and loss factors of the three-layered MRF beam structure were investigated by means of extended MM model. During the present study, the theoretical modeling of MRF sandwich beam based on shear configuration was performed. The existing model for viscoelastically damped composite three layer beams were examined for their applicability to MRF sandwich beams.

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2 The analytical model of the MRF

In order to predict the behavior of a structural material subjected to dynamic loading, an analytical model must be capable of representing the typical material characteristics and adequately describing the dynamic behavior. From the rheological findings it was observed that MRF exhibit linear rheological behavior similar to many common viscoelastic materials[5, 12]. Based on this similarity in material behavior a modification of existing model used to viscoelastically damped sandwich structures was proposed. Making use of Boltzmann's principle of superposition, linear viscoelastic material's constitutive relationship is presented as[2, 4]:

$$\sigma(x,t) = \int_0^1 \mu(t,\tau)\varepsilon(\tau)\,\mathrm{d}\tau \tag{1}$$

where

$$\mu(t,\tau) = \chi(t,\tau)\delta(t-\tau) - H(t-\tau)\frac{\mathrm{d}\chi(t,\tau)}{\mathrm{d}\tau} = -\frac{\mathrm{d}H(t-\tau\chi(t,\tau))}{\mathrm{d}\tau},$$

$H(t)$ is Heaviside's function.
Carrying on Fourier transform to equation (1), yields

$$\bar{\sigma}(\omega) = \bar{\mu}(\omega)\bar{\varepsilon}(\omega). \tag{2}$$

A bar over a variable represents the Fourier transform of the variable. $\bar{\mu}(\omega)$ is the complex moduli of MRF. Let $\bar{\mu}(\omega) = G(\omega)$, hence

$$G(\omega) = G'(\omega) + iG''(\omega) \tag{3}$$

where $G'(\omega)$ is the storage modulus and $G''(\omega)$ is the loss modulus.

$$G'(\omega) = \chi(\infty) + \omega\int_0^\infty [\chi(t) - \chi(\infty)]\sin\omega t\,\mathrm{d}t,$$

$$G''(\omega) = \omega\int_0^\infty [\chi(t) - \chi(\infty)]\cos\omega t\,\mathrm{d}t\,.$$

According to MRF's rheological studies[2], the shear stress-strain relation is analyzed in three regimes such as pre-yield, yield and post-yield regimes. These behaviors are illustrated in Fig 1. The MRF pre-yield regime is modeled by a linear viscoelastic model.

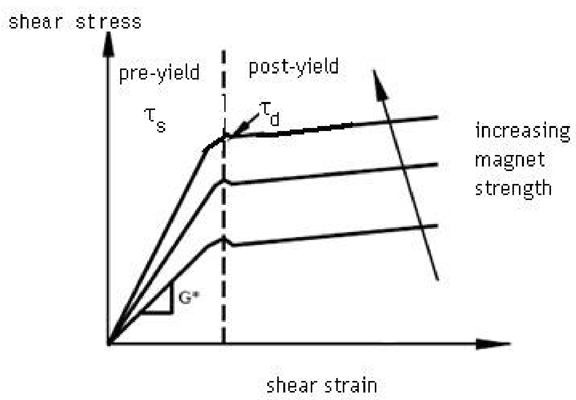

Fig. 1. Mrf's shear stress vs shear strain

$$\tau = G^* \gamma \tag{4}$$

where τ is shear stress, γ is shear strain, and G^* is the complex shear modulus represented in the form

$$G^* = G' + iG''. \tag{5}$$

In this study, the relationship between G',G'' and the applied magnetic field is expressed by experimentally identified relation by Qing Sun[9]. These relationships are given in equations (6):

$$\begin{aligned} G'(B) &= 3.11 \times 10^{-7} B^2 + 3.56 \times 10^{-4} B + 5.78 \times 10^{-1} \\ G''(B) &= 3.47 \times 10^{-9} B^2 + 3.85 \times 10^{-6} B + 6.31 \times 10^{-3} \end{aligned} \tag{6}$$

where B (oersted) is the magnetic induction.

3 The extended MM model of MRF sandwich beams

The smart composites sandwich beam is shown in Fig. 2. It consists of three layers, the upper and lower layers are elastic materials, the core is MRF. The simplified expression of the classical differential equation of motion for a beam structure subjected to damped flexural vibration is written as

$$\frac{\partial^6 w}{\partial x^6} - g(1+Y)\frac{\partial^4 w}{\partial x^4} + \frac{m(x)}{D_t}\left[\frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right] = -\frac{g f(x,t)}{D_t} \tag{7}$$

where
$g = \frac{G^*}{h_2}[\frac{1}{E_1 h_1} + \frac{1}{E_3 h_3}]$ is the complex shear parameter,
$Y = \frac{d^2}{D_t}[\frac{1}{E_1 h_1} + \frac{1}{E_3 h_3}]^{-1}$ is the geometric parameter,
$D_t = \frac{1}{12}(E_1 h_1^3 + E_1 h_1^3)$ is the flexural rigidity,
where $G^* = G' + iG'' = G'(1 + i\beta)$ the complex shear modulus of MRF, E_i is Young's modulus of the ith layer, h_i is the height of the ith layer, d is the distance between two elastic layers, $m(x)$ is the mass of the beam per unit length, $f(x,t)$ is the external force.

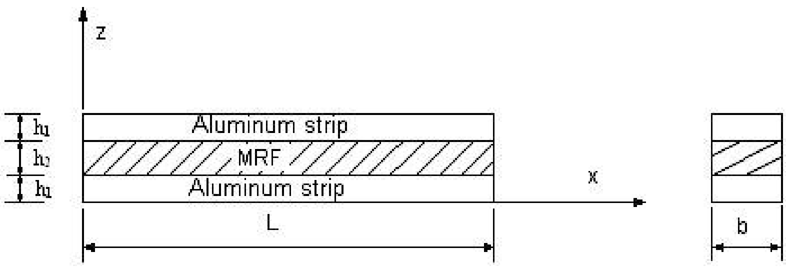

Fig. 2. Configurations of mrf sandwich beam

The assumptions considered in this model are listed as follows, (1) No slipping is assumed between the elastic layers and the MRF layer, (2) all three layers experience the same transverse displacement, and (3) no normal stresses in the MRF layer and no shear strains in the elastic layers exist.

If $f(x,t)$ is a harmonic forcing function and can be expressed in the form of: $F_0 e^{i\omega t}$, considering the simply supported boundary conditions for a finite length sandwich beam, the solution to equation (7) will be in the form

$$w(x,t) = W(x)e^{i\omega t}. \tag{8}$$

Substituting this expression into equation (7) , yields

$$\frac{\mathrm{d}^6 w}{\mathrm{d}x^6} - g(1+Y)\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} - \omega_n^2 \frac{m(x)}{D_t}\left[\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} - gw\right] = F_0. \tag{9}$$

For simply supported boundaries (x=0, l)

$$\text{transverse displacement} \qquad w = 0$$

$$\text{bending moment} \qquad M = EI\frac{\partial^2 w}{\partial x^2}$$

then the mode shape reduces to

$$W(x) = A\sin(\lambda x), \quad \lambda = \frac{n\pi}{l} \quad (n = 1, 2, 3\cdots). \tag{10}$$

When $f(x,t) = 0$, substituting this expression into equation (9), the expression for the natural frequency, loss factor will be

$$\omega_n = \lambda^2\sqrt{\frac{\lambda^4 + 2\lambda^2 g + \lambda^2 gY + g^2(1+Y)(1+\beta^2)}{(\lambda^2+g)^2 + g^2\beta^2}}, \tag{11}$$

$$\eta = \frac{\lambda^2\beta gY}{\lambda^4 + 2\lambda^2 g + 2g^2 Y + g^2(1+Y)(1+\beta^2)}. \tag{12}$$

And transverse response of the beam will be

$$w(x,t) = \sum_1^\infty \sin\frac{n\pi x}{l}\sin\frac{n\pi a}{l}\frac{F_0 e^{-i\varphi}e^{i\omega t}}{\sqrt{(\omega_n^2-\omega^2)^2 + (\omega_n^2\beta)^2}} \tag{13}$$

where a is the location of the external force, ω is the frequency of the external force. $\tan\varphi = \frac{\omega_n^2}{\omega_n^2-\omega^2}$. Thus, the extended model is capable of determining natural frequencies, mode shapes and transverse vibration response at a point on the MRF sandwich beam for an applied point force at a specified location.

4 Numerical example

A uniform symmetrical three-layer MRF sandwich beam with simply-supported conditions is taken as a numerical example. Its dimensions and material properties are as follows:

For the upper and lower elastic face sheets

Young' s modulus	$E = 70\times10^9$N/m^2
Density	$\rho_1 = \rho_3 = 2800$kg/m^3
Thickness	$h_1 = h_3 = 0.9$mm

For the MRF (KD1) layer

Shear modulus	G^* is expressed in equations (5,6)
Density	$\rho_2 = 3450$kg/m^3
Thickness	$h_2 = 1.0$mm

For the whole beam

Length	$l = 380$mm
Width	$b = 29$mm

The results of the example are illustrated in Fig. 3 and Table 1, Table 2, Table 3.

Fig. 3 shows the effect of the magnetic field on the vibration amplitude of the MRF sandwich beam. In the figure, as the magnetic field strength increases, the vibration amplitude of each mode decreases and natural frequencies are shifted to higher lever. Table 1, 2 and 3 illustrate the natural frequency and loss factor variations of up to four modes. As can be seen, the natural frequencies and loss factors are shifted to higher lever when the applied magnetic field increases.

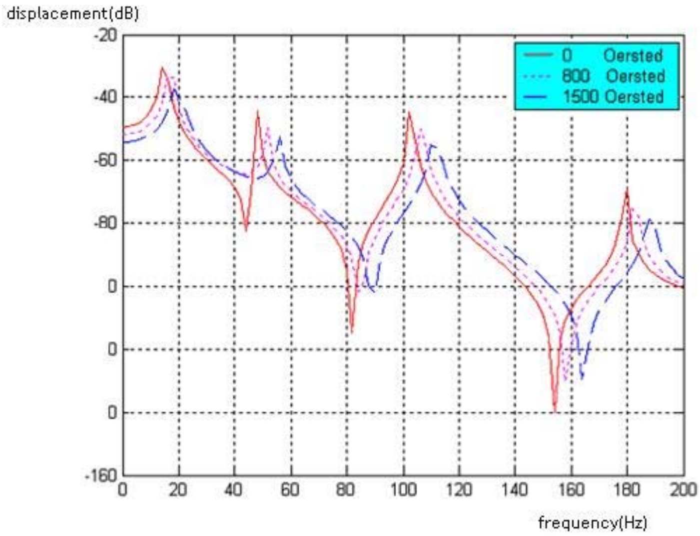

Fig. 3. Frequency response of mrf sandwich beam

Table 1. Comparison of numerical results with ref. [**?**] (0 oersted)

mode number	results of the example		experimental results of Ref. [10]	
	natural frequency	loss factor	natural frequency	loss factor
1	13.0571	0.00481	16.8514	0.00550
2	46.9600	0.00202	50.2599	0.00251
3	101.1841	0.00104	104.8868	0.00150
4	177.3417	0.00061	181.1732	0.00082

Table 2. Comparison of numerical results with ref. [**?**] (800 oersted)

mode number	results of the example		experimental results of Ref. [10]	
	natural frequency	loss factor	natural frequency	loss factor
1	15.5150	0.00531	19.0049	0.00651
2	50.0874	0.00284	53.4577	0.00382
3	105.4406	0.00152	108.4046	0.00243
4	181.3658	0.00100	184.8655	0.00112

Table 3. Comparison of numerical results with ref. [**?**] (1500 oersted)

mode number	results of the example		experimental results of Ref. [10]	
	natural frequency	loss factor	natural frequency	loss factor
1	18.3874	0.00601	21.4702	0.00722
2	54.7759	0.00400	61.7857	0.00552
3	111.2740	0.00232	113.4247	0.00340
4	186.6143	0.00143	190.2616	0.00254

5 Conclusions

In this study, vibration characteristics of a sandwich beam with smart composites—MRF have been investigated. From the findings of the analysis, it is observed that MRF presents vibration control capabilities. Vibration amplitudes are decreased with increasing in the applied magnetic field for MRF. Furthermore, the natural frequencies and loss factors are shifted to higher lever when the applied magnetic field increases.

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