ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 2 (2006) No. 3, pp. 186-189

Mesh-less model proposed for the simulation of running processes of cars

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(*Received September 17 2005, accepted November 3 2005*)

Abstract. Traditional methods have their shortcomings in simulating running processes of cars. A cellular automata model has calculation difficulties when the maximum speeds of the cars are different from each other or the road is very long. A mesh-less model is proposed. It assumes behind each car there is an attaching tail in horizontal direction. Name this kind of tail as imaginary tail. The target speed of a car is determined by the widths of the tails side by side with it, the width of the road and other local environment conditions. The target speed calculated in this way is not continuous. The deflection curve of a built-in beam is used to make the speeds continuous. The new simulation model has been used for running processes of cars with different maximum speeds. The simulation results showed that the new model could fit the complicate dynamic characteristics of the running processes of cars well.

Keywords: running process of cars, simulation, mesh-less model, imaginary tail, cellular automaton

1 Introduction

The simulation calculation of the speeds and places of running cars in any time is a challenging problem. Traditional methods still have lots of shortcomings. Cellular automaton based method is the most popular method. In a cellular automaton model, the road should be separated into a series of grids artificially[3]. The accelerating rule and decelerating rule are assigned and the speed of a car is determined according to the condition of the grids before it. If the restrain condition is violated, the speed is re-calculated[4, 5]. Cellular automaton models have been used successfully on getting the relationship between the average speed and the density of a traffic flow[2, 7, 8]. Yet one of the characteristics of this kind of model is that it is discrete both in space and time. This causes troubles when the length of the road or the considered time is very long. In the case that lane changing exists, a 2D cellular automaton model should be used to calculate the traverse speed of cars. This makes the calculation complicated. Actually, a cellular automaton could not work properly when the maximum speeds of the cars are different from one another.

A driver determines his operation according to the local environment. The speed of his car is a continuous piecewise smooth function of time. This mechanism is somewhat different from that of a cellular automaton model.

A lot of other kind car-following models have been studied[1, 6]. Yet each of them is not an union model which could involve the overtaking process.

2 Imaginary tail, target speed and real speed

A new concept—-target speed is presented here. The target speed of a car is calculated according to the local environment. It is not continuous. The deflection curve of a built-in beam will be used to make it continuous.

Another new concept, imaginary tail, is also presented here. Suppose there is an imaginary attaching tail behind each car. The shape of an imaginary tail is supposed to similar to that of the right part of the deflection curve of a built-in beam. An imaginary tail could restrain the speed of a following car gradually. For the i-th car, the width of the attaching imaginary tail could be expressed as

$$t_i = b_i \left\{ 1 - \left[3 \left(1 - \frac{\xi_i}{L_i} \right)^2 - 2 \left(1 - \frac{\xi_i}{L_i} \right)^3 \right] \right\}. \tag{1}$$

Here L_i is the length of the imaginary tail in longitudinal direction, ξ_i is the longitudinal distance between the car and one point on the imaginary tail, b_i is the width of the i-th car. Actually, the value of b_i could be grater than its real value by a little. The shapes and lengths of the imaginary tails will remain unchanged bellow.

For any car, its target speed is calculated with following formula:

$$\tilde{v}_j = v_j^{(\max)} (1 - c_j) + c_j v_{Lead}. \tag{2}$$

Here

$$c_j = \frac{\sum\limits_{k=1}^{l} t_{jk}}{b_{Road} - b_{Trunc} - b_j} \tag{3}$$

is an interpolation coefficient. In formula (3), b_{Road} is the width of the road, $b_{Trunc} > 0$ is an abundant value, b_j is the width of j-th car, and t_{jk} are the widths of imaginary tails belonged to those cars which are ahead and are slower than car j,

$$t_{jk} = \begin{cases} 0, & \text{when } s_k - s_j > \text{tail length of vehicle } k, \\ b_k[3(1 - \frac{s_k - s_j}{L_k})^2 - 2(1 - \frac{s_k - s_j}{L_k})^3], & \text{in other case.} \end{cases} \tag{4}$$

Whenever a car surpasses another car, the numerator of corresponding formula (3) is lowered by a limited value t_{k_0}. And the target speed expressed as formula (2) reduces a limited value $t_{k_0}/(b_{Road} - b_{Trunc} - b_j)$ accordingly. Hence the target speed is not continuous. The real speed of a car is calculated as following. At the ridge time τ_0 when car j surpasses another car, denote its real speed as $v_i^{(0)}$. In the right neighborhood of τ_0, it has

$$c_j = \frac{\sum\limits_{k=2}^{l} t_{jk}}{b_{Road} - b_{Trunc} - b_j}. \tag{5}$$

It should be noted that the label k of above formula varies from 2 to l (not from 1 to l). The real speed is calculated as

$$v_j = v_j^{(0)} - (v_j^{(0)} - \tilde{v}_j) \left\{ 1 - \left[3(1 - \frac{t - \tau_0}{T_R})^2 - 2(1 - \frac{t - \tau_0}{T_R})^3 \right] \right\}. \tag{6}$$

Here T_R is the characteristic time. The deflection curve of a built-in beam is adopted in above formula for the continuous of the speed.

3 Examples

Several examples will be considered below.

Example 1. 10 running cars with different maximum speeds

Suppose there are 10 cars running on a highway. Their maximum speeds are 50m/s, 41.7m/s, 25m/s, 22.22m/s, 23.6m/s, 33.3m/s, 30.6m/s, 36.1m/s, 36.1m/s, 27.8m/s, 33.3m/s, 30.6m/s, 36.1m/s, 36.1m/s and 27.8m/s respectively. Their initial places are at 0m, 50m, 100m, 140m, 180m, 260m, 270m, 300m, 330m and 360m respectively. At the beginning, assign each of them an index. The indexes are 1, 2, 3, 4, 5, 6, 7, 8, 9, and

10 respectively. The width of the road is supposed to be 6m. The width of every car is supposed to be 2m. And the abundant value b_{Trunc}=0.5m. Each car has an imaginary tail of 45m. The parameter T_R used for each car is 2 second.

Take the length of time step as 0.5 second. Calculate the speeds of the cars with the new model just proposed. The speed curve of car 1 during 0 second $\sim$ 20 second is shown in Fig. 1.

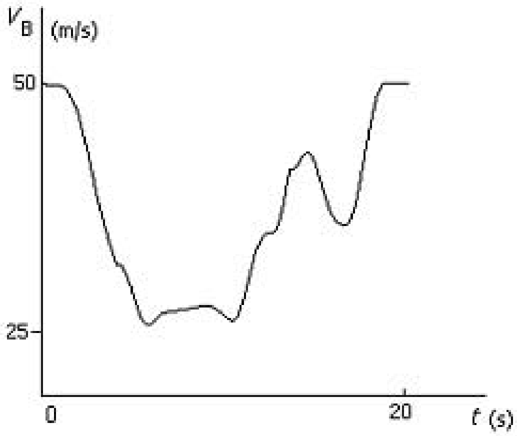

Fig. 1. The speed curve of car 1 in example 1

In above figure, the speed begins to decrease whenever car 1 touches a new imaginary tail. And the speed begins to increase whenever car 1 surpasses another car.

At the instant of t =132 sec, the arranging order of the cars reaches its stable state 4, 5, 3, 10, 7, 6, 8, 9, 2, 1. This arranging order remains unchanged since faster cars precede the slower cars in this order.

Example 2. Collecting-fees station encountered

Suppose the maximum speed of car 10 be zero. Its width is 4m. It stays at the place of 530m from the reference point. It could be thought as a collecting-fees station or a roadblock.

The simulation result is, all of the cars from number 1 to number 9 could not surpass car 10, and their speeds press on towards zero.

Example 3. Roadblock encountered abruptly by the leading running car

Still suppose the maximum speed of car 10 be zero. Let the width of car 10 be 2.5m. And the width of other cars is 2 m. Let b_{Trunc} =0m. Suppose car 10 stays at 490m from the origin point. Let the initial places of car 1 to car 9 be 0m, 150m, 255m, 360m, 365m, 400m, 420m, 450m and 480m respectively from the origin point respectively. Their maximum speeds equivalent their initial speeds. And their values are supposed to be 50m/s, 41.7m/s, 25m/s22.2m/s, 23.6m/s, 33.3m/s, 30.6m/s, and 36.1m/s respectively. The length of every imaginary tail is supposed to be 60m, the length of time step is chosen as 0.1sec.

Since the sum of the width of car 10 and that of another car is equivalent to the width of the road, the position order of the cars couldn't be changed. At the beginning car 9 nears to car 10 already. And the initial speed of car 8 is very fast. This case could be thought as a running car (car 10) stops abruptly in the middle of the road. Or it could be thought as the case that car 8 encounters a roadblock abruptly after turning a corner.

The simulation result shows that car 9 could not surpass car 10. This is because the imaginary tail of car 10 restrains the speed of car 9 naturally. Actually, the absolute value of the acceleration is very large. That means an accident has taken place.

Car 1, car 2, car 3, car 4, car 5, car 6, car 7 and car 8 also stopped at last. Their order remains unchanged. The corresponding imaginary tails restrains all speeds of them. The restrain conditions are satisfied automatically.

The speed curve of car 9 is showed as following.

Other examples have also been considered. These cases are: (1) the leading running car encounters a roadblock far ahead of; (2) The leading car runs slowly, the number of cars is very large and the road is very narrow; (3) The number of cars is very large and the road is quite wide that several cars might run side by side. All the simulation results of them showed that the new model works well.

4 Conclusions

The new model is a kind of mesh-less model. The imaginary tails reduces the speeds of following cars gradually. That makes the restrain conditions be satisfied automatically. If the width of the road is grater than the sum of the widths of any pair of cars, the faster car would surpass the slower car at last. If the width of the road is smaller than the sum of the widths of any pair of cars, the position order of the cars would remain unchanged. These trends could be seen in the simulation results.

The new model is a union model, which could deal with both car-following processes and overtaking processes. It is an 1D model. For each car, the number of the imaginary tails besides it is quite limited. Hence the computation time of the new model is in direct proportion to the total number of the cars. That means it could be used for problems in large scale. Actually, an example involving 7000 cars is simulated. The computation time for each simulation time step is about 2.8 sec. It is very short.

The new model makes full use of the deflection curve of a built-in beam, in this way the mechanism of real processes could be revealed simply and clearly.

Consider the case of two cars. During the period that the faster car is behind the slower car, the speed curve of the faster car is similar to the deflection curve of a built-in beam. But it is not expressed as a deflection curve of a built-in beam. The former has a high order nonlinear than the later has.

The new model could be adopted for the simulation of accidents. When the acceleration of a car is larger than given critical value, it could be thought, as there is an accident.

The delayed time of each car j is the ratio of a surface to its maximum speed. This surface corresponds to the region enclosed by the speed curve and the horizontal line

$$v_j = v_j^{(\max)}.$$

Thus the delayed time of any car could be calculated through its speed curve with integral method.

From the above analysis, it could be seen that the new model has a good popularization prospect.

References

[1] S. Q. Dai, L. Lei, L. Y. Dong. Analysis of traffic flow at intersections near ramps of overhanging freeways. *Acta Mechanica Sinica*, 2003, **35**(5): 513–518. (in chinese).

[2] X. Y. Lu, M. R. Liu, L. J. Kong. Theoretical analysis and computer experiments for 1d random traffic flow models. *Acta Physica Sinica*, 1998, **47**(11): 1761–1768. (in chinese).

[3] T. Nagatani. Kinetic clustering and jamming transitions in a car-following model for bus route. *Physica A*, 2000, **287**(1-2): 302–312.

[4] K. Nagel, P. Wagner, R. Woesler. Still flowing: approaches to traffic flow and traffic jam modeling. *Operations Research*, 2003, **51**(5): 681–710.

[5] L. Roters, K. D. Usadel, S. Lubeck. Critical behavior of a traffic flow model. *Physical Review E*, 1999, **59**(3): 2672–2676.

[6] S. Tzila. How should an autonomous car overtake a slower moving car:design and analysis of an optimal trajectory. *IEEE Transactions on Automatic Control*, 2004, **49**(4): 607–610.

[7] B. H. Wang, P. M. Hui, B. B. Hu, L. Wang. The asymptotic steady states of deterministic one-dimensional traffic flow models. *Physica B*, 2000, **279**(1-3): 237–239.

[8] B. H. Wang, L. Wang, P. M. Hui, B. B. Hu. The gradual accelerating traffic flow cellular automaton model in which only high speed car can be delayed. *Acta Physica Sinica*, 2000, **49**(10): 1926–1931. (in chinese).