ISSN 1 746-7233, England, UK
**World Journal of Modelling and Simulation**
Vol. 2 (2006) No. 3, pp. 182-185

# Nonlinear dynamics model and application in automobile logistics

Qiu Wu* , Guanxin Yao

Automobile and Transport Engineering College, Jiangsu University, Zhenjiang 212013, China

(*Received October 25 2005, Accepted December 1 2005*)

**Abstract.** According to the complexity of automobile logistics, we take one company's automobile logistics as sample to analyze dynamics characteristics of logistic system and to set up system model by applying nonlinear dynamics theory and measure. Also, we can prove that it is applicable to study automobile logistics by this model according to the analysis of the model's chaotic dynamics characteristics. This will be reliable scientific basis for China's automobile logistics.

**Keywords:** logistic system, automobile logistic, nonlinear dynamics, chaotic dynamics

## 1 Introduction

Recent years, with China's leading effect on automobile market increase, the Asian automobile market keeps on the increasing rate of over 10%[7]. At the same time, with quick development of China's automobile industry, the demands for automobile logistics increase a lot. Due to its high requirement and difficulties in operation, the integrated development level of automobile logistics is much laggard in China[2, 5, 6]. According to the statistics on automobile business between January and September in 2004, we can see that totally we have produced 320,000 automobiles, sold out 3,070,000 and kept 130,000 in stock, among which, the production for the car was 30,000, sale of it is 1,340,000 and 90,000 were in stock. From the figure, we know that the cars in stock increased 30,000 compared with those two months ago. By analyzing the car stock structure, we can see that the increased stock is mainly the unsalable cars. This situation shows that the relationship between automobile demand and supply is changing, with opening market further, which will strengthen market competition and decrease the automobile-selling profit. While, the automobile logistics as the third profit source becomes more and more important. If we can predict correctly for the automobile logistics in the $11^{th}$ and $12^{th}$ Five-year Developing Plan, we can offer a reliable scientific foundation for the development of China's automobile logistics[3, 4, 8].

With so many factors having the influence on automobile logistics, the system is a complicated nonlinear dynamic system. Thus the prediction method in usual statistics cannot make correct prediction on automobile logistics. To set up one complete model for automobile logistics system, we not only need to collect and study the datum and tables which can reflect the outside situation of the system, but also need to study the information on system structure. To make one model which can factually reflect the system's essential characteristics, we need explore the system by observing those immeasurable consequences to connect the system's dynamic changes to its inner feedback structure. Thus, we can get the model which can really reflect the system's essence. After getting the correct structure of the model, it can correctly reflect system's essence and operation characteristics, even the datum used are not precise enough. The chaotic dynamic model set up for automobile logistics in the article can describe the automobile logistics well.

There are 4 sections in the article. Section 1 is foreword. Section 2 sets up dynamic function for automobile logistics. Section 3 analyses the dynamic characteristics of the function, which is a nonlinear discrete

* Corresponding author. Tel.: +86-511-8780576; fax: +86-511-8791467.
*E-mail address*: qcjx@ujs.edu.cn.

dynamic system different from usual logistics system. Section 4 is the conclusion to illuminate the healthy, sustainable and steady development of automobile logistics system which can be realized by proper parameter adjustment through strengthening government's macro-control, building up infrastructure needed by automobile logistics, setting up national information platform for automobile logistics and supplying information on automobile logistics.

## 2 Dynamic function for automobile logistics

It is a complicated system to administrate the automobile logistics in one month. Such as, the automobile logistics will be heavily influenced by market, which makes it uncontrollable. Automobile logistics should emphasize on Customer-focus. It is very difficult to control the stock for automobile logistics. By analyzing system working flow, observing changing relationship among factors in the system, we can see that the monthly production volume is the key factor to influence automobile logistics. According to its monthly stock, automobile logistics, the $3^{rd}$ profit origin, will set up one scheme with close relationship with several factors, including the infrastructure construction for automobile logistics, the information transfer capability of national automobile logistics, the cost of automobile logistics, the government's macro-control strength, service level of automobile logistics companies, etc. Thus, the precise analysis and calculation is very difficult to be realized. Observing the consequence among all factors influencing transport volume by system dynamic method, we will find that there is one positive feedback effect existing. Excluding other factors, we set up one function for automobile logistics:

$$E = AK^a. \tag{1}$$

Here, $E$ is the link index for monthly automobile production. $K$ is the link index for monthly automobile stock. $a$ is the output elasticity index of fixed assets investment. $A$ is the coefficient of integrated factors. The coefficient here is the relativity showing general quantitative change of complicated social economic phenomena. The index can be classified as individual index and total index according to different objects, or can be classified as quantitative index and qualitative index by different economic index character, or can be classified as fixed index and link index by different comparison benchmark period. The link index mentioned here is a series of index based on the benchmark period before report time and it is listed in the index progression. It is one key factor to analyze the changing orientation and degree of general complicated economic phenomenon and the all factors' influence on economic development.

Let $\alpha$ is the upper limit for monthly automobile stock-$K$'s increase rate. Let $\beta$ is the influence factor on $K$'s increase rate by output of unit asset investment. By analyzing the relationship among $\alpha$, $\beta$, $K$ and $E$, it shows that $\alpha$-the upper limit is the sum of monthly automobile stock-$K$'s increase rate and the influence factor on $K$'s increase rate by output of unit asset investment. Thus, the Haavelmo Increasing Model with consecutive form shows as follows:

$$\frac{\dot{K}}{K} = \alpha - \frac{\beta K}{E}, \quad \alpha, \beta > 0. \tag{2}$$

Put function (1) into function (2), we can have the consecutive system for automobile logistics model as follows:

$$\dot{K} = K\left(\alpha - \frac{\beta K^{1-a}}{A}\right). \tag{3}$$

Calculating differential equation, we get:

$$K(t) = \left[\frac{1}{A}\left(\left(AK(0)^{a-2} - \frac{\beta}{\alpha}\right)e^{\alpha(1-a)^t} + \frac{\beta}{\alpha}\right)\right]^{\frac{1}{a-1}}.$$

Analyzing the result of (3), when $K(0) > (<)\left(\frac{\alpha A}{\beta}\right)^{\frac{1}{2-a}}$, $K$ and $E$ monotonously decrease or increase, and approaching their exclusive steady situation value $K_s = \left(\frac{\alpha A}{\beta}\right)^{\frac{1}{2-a}}$ and $E_s = A\left(\frac{\alpha A}{\beta}\right)^{\frac{a}{1-a}}$.

Now, replace differential coefficient by difference, we get:

$$E_t = AK_t^a, \tag{4}$$

$$\frac{K_{t+1} - K_t}{K_t} = \alpha - \frac{\beta K_t}{E_t}. \tag{5}$$

Then, we can have discrete form of automobile logistic system model as:

$$K_{t+1} = K_t \left[ (1+\alpha) - \frac{\beta K_t^{1-a}}{A} \right]. \tag{6}$$

Let $K_{t+1} = \left(\frac{A(1+\alpha)}{\beta}\right)^{\frac{1}{1-a}} x_t$, and put it into (6), we get:

$$x_{t+1} = (1+\alpha)\, x_t \left(1 - x_t^{1-a}\right) \qquad (0 < \alpha < \alpha^*). \tag{7}$$

Among which $\alpha^* = \left[\left(\frac{1}{2-a}\right)^{\frac{1}{1-a}} \left(1 - \frac{1}{2-a}\right)\right]^{-1} - 1$. Formula (7) is the discrete dynamic system for automobile logistic system model. This is a kind of nonlinear discrete dynamic system[1]. It is different from usual logistic system[9], because it is more complicated and be influenced by parameter $a$. When $a = 1$, it is usual Logistic equation. While common situation always shows $a \neq 1$. This article analyzes the dynamics characteristic of system (7) and studies the system's effect on automobile logistics.

## 3 Dynamics characteristics of the function in automobile logistics

Let $\lambda = 1 + \alpha, 1 - \alpha = \frac{p}{q}$, among which $p, q$ are comprise positive integrals. Then formula (7) can be presented as $x_n = \lambda x_n \left(1 - x_n^{\frac{q}{p}}\right)$. Let $f(x) = \lambda x \left(1 - x^{\frac{q}{p}}\right)$, we got two stable points of the system: $x_0 = 0$ and $x_1 = \left(1 - \frac{1}{\lambda}\right)^{\frac{p}{q}}$. While $f'(x) = \lambda - \lambda\left(1 - \frac{q}{p}\right) x^{\frac{q}{p}}$, when $x_0 = 0$, $|f'(x_0)| = \lambda > 1$. So $x_0 = 0$ is the repellor of the system.

When $x_1 = \left(1 - \frac{1}{\lambda}\right)^{\frac{p}{q}}$, then $|f'(x_0)| = \left|1 + \frac{q}{p}\lambda - \frac{q}{p}\right|$. While when $\lambda < 1 + \frac{2p}{q}$, $x_1 = \left(1 - \frac{1}{\lambda}\right)^{\frac{p}{q}}$ is the attractor of the system. When $\lambda > 1 + \frac{2p}{q}$, $x_1$ will lose its stability and 2 period point appear in system. With the increase of $\lambda$, the system will be bifurcated from 2 period to 4 period to 8 period to 16 period, even more period doubling bifurcation. When $\lambda$ increase further to $\lambda_\infty$, the system will be chaos, as shown in Fig. 1.

When $\lambda$ increase again to $\lambda_{-1}$, the system will change from chaos to period doubling bifurcation, which means the first inverted bifurcation appears. Shown in Fig. 2.

When $\lambda$ increase continuously, the system changes to chaos again from inverted bifurcation. Shown in Fig. 3.

With $\lambda$'s continuous increase from $1 + \frac{2p}{q}$ to $\left(1 + \frac{q}{p}\right)^{\frac{p}{q}} \left(1 + \frac{p}{q}\right)$, the system changes between chaos and inverted bifurcation continuously. The chaos appears in system with parameter change.

Next analysis for system (7) is based on elasticity coefficient $a$ on the output of fixed assets investment. When $0 < \alpha < \frac{2}{1-a}$, the system is steady and the automobile logistic system is in benign circulate. When $\frac{2}{1-a} < \alpha < \alpha_c$, system is in period doubling bifurcation situation ($\alpha_c$ is the critical point of system entering chaos). At this time, the increase rate of monthly automobile stock link index needs to be adjusted, such as adjust monthly automobile production link index and monthly automobile stock link index or strengthening infrastructure construction of automobile logistics and set up national information platform for it and supply related information to make the system stable. When $\alpha_c < \alpha < \alpha^*$, system is in chaos. To make sure the healthy, sustainable development of China's automobile logistics, we need not only the former mentioned measures, but also need to strengthen governmental macro-control and to set up general blue print of automobile logistics refer to advanced international experience. Thus, in actual application, we need adjust behavior according to self condition and outside circumstance to ensure reasonable and smooth operation of automobile logistic system.

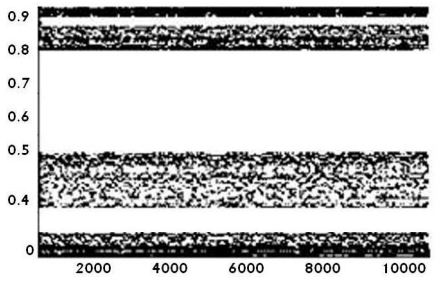


**Fig. 1.** Chaos

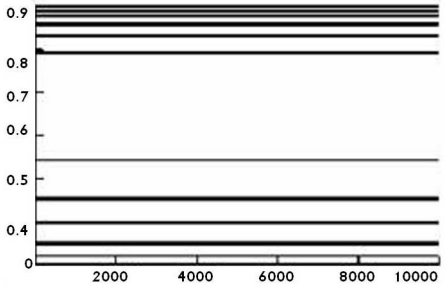


**Fig. 2.** Inverted bifurcation

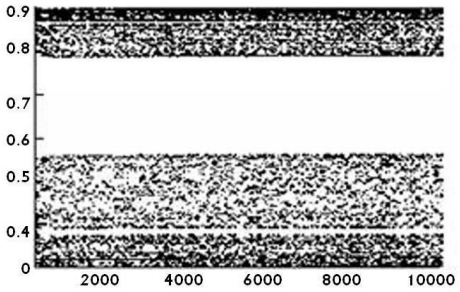


**Fig. 3.** Chaos

## 4 Conclusions

This article sets up one nonlinear model for a kind of automobile logistic system and puts system dynamics into the application for China's automobile logistic system. This method adequately considers the system's dynamic behavior characteristics and deepens the recognition of the essence of automobile logistic system. It also points out that in actual operation, we can achieve healthy, sustsainable and steady development of automobile logistic system by proper parameter adjustment through the measures as strengthening governmental macro-control, increase infrastructure construction for automobile logistics system, setting up national information platform, supplying information for automobile logistics system, etc.

## References

[1] S. H. Chen, J. N. Lu. *Introduction of Chaos Dynamics*, 1st edn. Wuhan Water Electricity University Press, 1998.

[2] F. J. Gao, X. Q. Zhou. The application of system dynamics in scattered cargo logistic information system. *Modern Electricity*, 1997, **14**(2): 41–45.

[3] X. L. Wang, X. D. Zhou, W. J. Zhang. Dynamic Alliance-Logistics administration new model in 21 century. *Logistics Science and Technology*, 2000, **2**: 33–34.

[4] X. Y. Wang, H. Wang. The organization mode of Chevron company's in-time supply Logistics. *International Logistics*, 1996, **6**: 34–36.

[5] Z. Wang, L. P. Zhang. The current situation and development of China's automobile logistics. *Modern Logistics*, 1999, **9**: 41–42.

[6] H. X. Yang, Z. Y. Liu, M. Y. Wang. The study on optimization coefficient system of automobile logistic companies. *Shanghai Engineering Technology University Journal*, 2004, **18**(3): 230–235.

[7] J. M. Zhang. The strategy study on China's automobile logistics development. *Logitics & Material Handling*, 2004, **1**: 80–83.

[8] Y. X. Zhang, Y. J. Wang. The nonlinear dynamic model for forecast of coal demand increase rate. *Xiangtan Mining College Journal*, 1997, **2**: 1–6.

[9] R. F. Zhou, X. J. Liu, J. Yu. *The Principle of Statistics*, 1st edn. 2003.