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Fluctuating consumption behavior under uncertainty *

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Abstract. Uncertainty is one of important factors which influence household's consumption behavior in different way. In this paper, some studies on the uncertainty issue are presented after summarizing the classical theory. In the frame of new classic economics theory about consumption, our research focuses on the fluctuant consumption behavior under uncertainty. As a result, we have pointed out the direction to reduce impression of uncertainty, and given some instructive advices on government policy.

Keywords: uncertainty, consumption, utility function

1 Introduction

In an economic system the uncertainty and the risk is different at root even though they resemble each other in appearance. The uncertainty is inherent property of any economic system. and it's influence on economic activities could not be predicted and measured exactly. But the risk has the possibility which can be analyzed or estimated in a sense of statistics. Uncertainty emerges mainly in two respects of natural environment and the social. The natural environment uncertainty can not be controlled by manpower, such as weather change, natural disaster etc. The uncertainty in social environment is the behavioral result in human community, that is closely related with international environment, political system, economic mode, macro control policy, international trade structure and game behavior on the market, etc[4,8]. It can be changed by mankind, but the changed consequence is very hard to be estimated precisely, especially for the uncertainty outside the economic system, such as unsteady government, unbalance of international trade, the consciousness of country, the turbulence of political situation with latent military clash, etc.The consumption problem has been a very worthy subject for economics and management researchers. At one respect, the consumption system formed with lots of individual consumer is open, gigantic and complicated, since the consumer individuals would be influenced by multi-level interactions and interrelationships; at another respect, some limits come from macroscopic factors in nature, politics and economy, etc. These uncertainties must have important influence over the consumptive behaviors of residents. In China current economy just goes upon a changeable stage thanks to high speed growth, system transfer and structural adjustment, therefore there is no mature economic mode but the possibility that some elements change dramatically, the uncertainty inside or outside of consumptive economy has stronger influence upon consumer behaviors than before[1,10]. This situation probably makes the research on consumption uncertainty tremendous value to both the country's macro-control improvement and enterprise development. As the limited space in this paper, only can we recount a part of our studies.

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2 Classical economics theory

In the 30's of twentieth century, L.R.knight, G. Myrdal, John Maynard Keynes, J.R.Hicks and some of other economists had studied on uncertainty or expectations. L.R.knight(1921) pointed out the essential distinction between uncertainty and risk, because the definition of risk is on the random cases which emerge at certain rate, while the random cases may occur at uncertain rate, so he emphasized that people has no way to foresee uncertainty by precise measure. His thought was inherited and developed by John Maynard Keyne, he considered that almost every economic decision making were made under uncertain conditions, and more important thing was to take the uncertainty as the basis of economics theory framework[12].

The economists after Keynes had proceeded with uncertainty research around the expectation theory, and they had got critical progress. In the truthful analysis of relations between consumer and uncertainty, Milton Friedman (1957) considered that expenditure inclination based on sustainable income can be the decreasing function of income uncertainty[3,8,9]. When consumer individuals found that the occupational risk increases, the inclination of save will increase more. This kind of motivation originates from the savings for uncertain future. Muse(1961), who took the uncertainty in full consideration in economic system, and issued the reasonable expectation theory; his main conclusion is that some unforeseen uncertain factors will lead a expectation deviation to the real valuation, expectancy as an endogenous variable which changes with the controlled variable in stochastic process and the party expectancy about variable will be changed while the conditional probability distribution changes. The expectancy theory by Muse and the adaptive theory by Cagan (1956) jointly developed economics methodology and they had an important role to the development of modern economics theory. Researches respectively focused on property combination theory by James Tobin (1958), and on information chaos and filtering theory by Robert Lucas (1973,1975) as well as on reasonable expectancy by Robert.Barro(1976) have improved our knowledge about uncertainty[5,11].

The uncertainty research had not been handled as same as the regular problem until in last 30 years while preventative savings theory profoundly enriched our knowledge of uncertainty problem due to those scholar's tribute come from every countries. Leland Yeager(1967) defined the preventative savings motivation as the additional savings arose from and aimed at preventing the shock with uncertain income in future. By the two-period-model, and another economist (1970) studied that how the two aspects of uncertainty of income and risk influenced the saving decision-making, he got a conclusion that increase of uncertainty about future income will push consumers not to increase expenditure but more savings. Then Miller(1976) and Frank Sibley (1975) led this study to a multi-period- model time. Five years later George Dantzig(1981) further studied the effects on government policy making by saver's preventative motivation. Burrhus Frederick Skinner(1988) and Blanchard(1989) emphasized that only the response of sustainable income wave to the preventative motivation was very critical. From 1989 to 1990 at least two economists pointed out that immoderate sensitivity, and immoderate smooth of consumption could be explained by preventative savings theory, therefore in future the researchers ought to focus on the relations between family wealth and uncertainty. Kimball(1990,1991) and Weil (1992) researched the relation between preventative motivation, risk-aversion-lag alternative. Kimball called the preventive saving as "cautiousness" motivation, distinguished absolute cautiousness and relative motivation, considered that savings motivation theory and risk evasion have similarity[9]. Deadon (1991) offered buffer stock model in which the influence of uncertainty to it was fully estimated.

3 The uncertainty's influence over consumption

In the research of consumption the usual method to deal with uncertainty is to use expectant utility to replace current utility, means to use function (1) to replace (2) as below:

$$V = E_t[\sum_t^T U(C_\tau)] = E[\sum_t^T U_t(C_t)|I_t], \tag{1}$$

$$V = U(C_1) + U(C_2) + ... + U(C_T). \tag{2}$$

The two above are the special description of the life-cycle utility function under the assumption that consumer's preference can be added up or separated overpass time. In function (1), E_t represents a consumer's expectation in time $t$during his limited life-cycle. $U(\cdot)$ represents the utility function. The right item emphasizes that E_t (the consumer's expectation in time t) is the mathematical expectation confined by I_t (usable information in time t). Although consumers choose different consumption level in different periods according to a certain consumption mode, they can not make the choices for the future, since they can not know certain information about their income, property profits and expenditure at the certain time in the future. So, it is unrealistic for consumers to determine their future consumption unless they have special needs. Therefore, for future consumption is uncertain in time t, a feasible method for a rational consumer is to maximize the expectant utility expressed in function [4].

For a consumer, there are limited future natural states s(s=1, 2, 3, ..., s) during his life-cycle. Observing from time t, we can find that different choices of future states constitute a tree-structure, each of the trees represents a future state from time t to T through the whole proceed. Using p_S to represent the probability connected with each future path, the expectant utility in function (1) can be described as:

$$V = \sum_{s=1}^{S} \sum_{\tau=t}^{T} p_s U(C_{\tau s}). \quad (3)$$

By doing this, expectant utility theory and overpass-time-separating theory together engender doubly addable preference form. This is the inevitable outcome that using overpass-time-additivity theory without abandoning expectant utility theory. The additivity in the meantime originated from both overpass-time-additivity and expectant utility means that the relation between the degree of overpass-time-alternative and risk-evasion is negative (Deadon, 1992). By instinct, time is what close contact with uncertainty. In an uncertain world, future consumption will take more risks than current consumption. However, disputes still exist. For example, Hall (1989) had criticized this statement, thought that the two aspects of overpass-time-alternative and risk-evasion were independent from each other in consumer preference[2,6,7].

There are uncertainty and risk on channels to the future. Different choices by consumers will lead to huge differences in final results of their life-cycle. Even having the same social-background, the same initial property and the same expectation to the future, people's consumption channel and final result will be different from each other under the gradual accumulation of uncertainty's influence in their lifetime.

We will choose the "representative consumers" to go on our analysis. "Representative consumers" who have limitedly rational expectation, live in an uncertain environment. We always apply expectation value to deal with uncertain problems in technology, i.e. when facing uncertain future, consumers optimize their programming of future behavior on the ground of reasonable expectation value of correlated variable.

In the frame of new classic economics theory about consumption, limited-rational economic human beings pursue the maximization of utility function. In the stipulation of usable resources, they optimize their consumption each period. The optimization of personal utility can be described as:

$$\begin{aligned} &\max_{\{C_s\}_{s=t}^{T}} E_t \left[\sum_{s=0}^{T-t} (\tfrac{1}{1+\delta})^s U(C_{t+s}) \right] \\ &s.t. \quad A_{t+1} = (1+r)A_r + Y_t - C_t. \end{aligned} \quad (4)$$

In the function above, E_t represents a consumer's expectation in time t during his limited life-cycle. δ represents the time-preference rate overpass-time (future utility can be described as a current value by discount rate δ). $U(\cdot)$ represents the utility function. We do not make any assumption to characteristic of utility function for the sake of the general. A_t, Y_t, C_t respectively represent the property, income and expenditure that consumer posses in time t during his life-cycle. r represents profit rate related to property (the simplest assumption of r is the rate of interest).

The solution of questions above is a formal function of consumer's initial income-level-sequence and initial property:

$$C_s^* = f(Y_s, Y_{s-1}, ..., Y_0, A_0). \quad (5)$$

We can define V^* as the value function of the utility expectation after adopting certain optimized tactics for convenience, namely:

$$V_t^* = E_t\left[\sum_{s=0}^{T-t}(\frac{1}{1+\delta})^s U(C_{t+s}^*)\right]. \tag{6}$$

As a common saying: plans always follow changes. It is unrealistic for consumers to directly make several consumption plans for the future, because of the existed uncertainties and the confinement brought by current information about future income and expenditure. So we consider an alternated consumption programming which overpasses two periods: if a deviation x exists between current and optimized consumption mode, we can revise it in the next period, means:

$$\hat{C}_t = C_t^* - x, x > 0; \quad \hat{C}_{t+1} = C_{t+1}^* + (1+r)x; \quad \hat{C}_{t+s} = C_{t+s}^*, s \geq 2.$$

Consumer adopts the consumption programming tactics above and forms a consumption-process series $\{\hat{C}_s\}_{s=t}^T$, so we get the corresponding value function $\hat{V}_t$:

$$\hat{V}_t = E_t\left[\sum_{s=0}^{T-t}(\frac{1}{1+\delta})^s U(\hat{C}_{t+s})\right]. \tag{7}$$

Obviously, $V_t^* \geq \hat{V}_t$ since V_t^* is the corresponding optimized value function of $\hat{V}_t$, so we can get:

$$\hat{V}_t^* - \hat{V}_t = U(C_t^*) - U(C_t^* - x) + E_t\left[\frac{1}{1+\delta}U(C_{t+1}^*)\right] - E_t\left[\frac{1}{1+\delta}U(C_{t+1}^* + (1+r)x)\right]. \tag{8}$$

If $x = 0$, then there is no deviation between optimized mode and current one, namely:

$$\frac{\partial[V_t^* - \hat{V}_t]}{\partial x}\mid_{x=0} = 0. \tag{9}$$

Then we can get:

$$U'(C_t^*) - \frac{1+r}{1+\delta}E_t[U'(C_{t+1}^*)] = 0. \tag{10}$$

So we get the Euler Formula:

$$U'(C_t^*) = \frac{1+r}{1+\delta}E_t\left[U'(C_{t+1}^*)\right]. \tag{11}$$

We can get the conclusion from the process above: if we decrease 1 unit for expenditure nowadays, we can increase $(1+r)$ unit consumption in the next period. If consumers have optimized their expenditure, then actual consumption equals the optimum consumption in this period. So we have:

$$E_t\left[U'(C_{t+1})\right] = \frac{1+\delta}{1+r}U'(C_t). \tag{12}$$

The description of actual consumption in the next period can be expressed as: an uncertain factor ε_{t+1} adds to the consumption expectation in the next period. Obviously:

$$U'(C_{t+1}) = E_t[U'(C_{t+1})] + \varepsilon_{t+1}. \tag{13}$$

Further more:

$$U'(C_{t+1}) = \frac{1+\delta}{1+r}U'(C_t) + \varepsilon_{t+1}. \tag{14}$$

If a consumer has known all information about his income and expenditure in the next period, we can conclude that $E_t[\varepsilon_{t+1}] = 0$. For ε_{t+1} is an uncertain factor of the next period's consumption expectation, which has no dependence relation to the current expenditure C_t in this period, we can deal function(14) with normal regression .

From functions above, we can see that the key to analyze the influence of uncertainty on consumption is how to choose the utility function. In fact, the curvature of utility function plays an important role in consumption theory, since it expresses consumers' attitudes to risk. For example, the Relative Risk-Evasion Coefficient in Arrow-Pratt Standard is defined by the consumption marginal utility elastic.

We will do some study on two kinds of utility function.

The first one is square utility function. The function can be expressed like:

$$U(C) = k_1 C - \frac{k_2}{2} C^2,$$

so:

$$U'(C) = k_1 - k_2 C.$$

Put it into the function above, we will have:

$$C_{t+1} = (\frac{1+\delta}{1+r} - 1)k_1 + \frac{1+\delta}{1+r} C_t + \frac{1}{k_2}\varepsilon_{t+1}. \tag{15}$$

Factor $\frac{1}{k_2}\varepsilon_{t+1}$ which equals uncertainty ε_{t+1} in form is used to describe uncertainty.

Then considering Hall's regression equation (1978) [2,6], then:

$$C_{t+1} = \beta_0 + \beta_1 C_t + \beta_2 Y_t + \varepsilon_{t+1}. \tag{16}$$

Function(15) and (16) can keep uniformity under the hypothesis below:

$$H_0 = \begin{cases} \beta_1 = \dfrac{1+\delta}{1+r} \\ \beta_2 = 0 \end{cases}. \tag{17}$$

Hall analyzed the experimental data, then got the results: $\hat{\beta}_1 = 1.02$(standard error 0.044), $\hat{\beta}_2 = -0.10$(standard error 0.032). Obviously, $\beta_1 \cong 1, \rightarrow \delta = r$, while β_2 is not important in statistics. Then he got the same description to the random walk hypothesis:

$$C_{t+1} = C_t + \varepsilon_{t+1}. \tag{18}$$

It means that uncertainty exists in the next period's expenditure while current consumption makes inertial effect on it.

What we really interested in is the proportional part which the uncertainty account for in the next period's consumption, or, whether uncertainty plays as a revised random factor (relatively smaller in term of data quantized level, its influence on background noise level), or as an important one determining the trend of future consumption (relatively bigger in term of data quantized level, its influence on drag force level).

If uncertainty's influence is on a background noise level, future consumption can be anticipated mostly. If it is an important one on drag force level, the consumptive expectation in several coming periods will be unpredicted under the accumulation of uncertainty's influence.

The second one is equal-elastic utility function. The function can be expressed like:

$$U(C) = \frac{C^{1-\rho}}{1-\rho},$$

$\rho(\rho > 0)$ represents the relative risk-evasion coefficient,

$$\rho = -\frac{C \cdot U''(C)}{U'(C)}. \tag{19}$$

Obviously, the relative risk-evasion coefficient is: CRRA(Constant Relative Risk Aversion). So, $U'(C) = C^{-\rho}$. According to the Euler Formula,

$$C_t^{-\rho} = \frac{1+r}{1+\delta} E_t[C_{t+1}^{-\rho}], \tag{20}$$

Described by exponential form:

$$C_t^{-\rho} = \frac{1+r}{1+\delta} E_t[e^{-\rho \ln C_{t+1}}], \tag{21}$$

Considering the logarithmic distribution of most consumption is normal distribution, when x under this distribution, it exists: $E[e^x] = e^{\mu} e^{V/2}$. So we have:

$$E[e^{-\rho \ln C_{t+1}}] = e^{-\rho E_t[\ln C_{t+1}]} e^{\rho^2 \sigma^2/2}.$$

Therefore, the Euler Formula can be rewritten like this:

$$C_t^{-\rho} = \frac{1+r}{1+\delta} e^{-\rho E_t[\ln C_{t+1}]} e^{\rho^2 \sigma^2/2}. \tag{22}$$

Taking its both sides into logarithmic forms:

$$-\rho \ln C_t = \ln(1+r) - \ln(1+\delta) - \rho E_t[\ln C_{t+1}] + \rho^2 \frac{\sigma^2}{2} \tag{23}$$

Then we have:

$$E_t[\ln C_{t+1}] = \ln C_t + \frac{\ln(1+r) - \ln(1+\delta)}{\rho} + \rho \frac{\sigma^2}{2} \tag{24}$$

In function(24), the constant which is the sum of the 2^{nd} and 3^{rd} items, represents that expenditure expectation in logarithmic forms takes a constant change from one period to another. Therefore, the anticipant expenditure increase is:

$$E_t[\ln C_{t+1} - \ln C_t] = \frac{\ln(1+r) - \ln(1+\delta)}{\rho} + \rho \frac{\sigma^2}{2}. \tag{25}$$

So we get:

$$\frac{\partial E_t[\ln C_{t+1} - \ln C_t]}{\partial r} = \frac{1}{\rho(1+r)} > 0, \tag{26}$$

$$\frac{\partial E_t[\ln C_{t+1} - \ln C_t]}{\partial \sigma^2} = \frac{\rho}{2} > 0, \tag{27}$$

$$\frac{\partial E_t[\ln C_{t+1} - \ln C_t]}{\partial |\sigma|} = \rho |\sigma|. \tag{28}$$

Obviously, we can get following conclusions:

If the interest rate increases, the expectation of expenditure will increase.

Given a certain r, the smaller ρ is, the bigger the expectant extent to expenditure will be.

If σ^2 grow, the expectation of expenditure increase does the same.

Generally, increasing uncertainty lead a bigger expectation on expenditure increasing, it also means an increase on savings in this period. On this point of view, it is displayed as preventative savings, namely, when facing a greater uncertainty, consumers will increase their current savings. Expectation on expenditure increasing which is sensitive to uncertainty, directly relates to two factors: the degree of consumers' risk aversion and uncertainty.

4 Methods on measuring uncertainty

Consumers' limited natural states S= (1, 2, . . . , s), are what experience in both uncertain natural and social environment. How to measure uncertainty and how much it is? This is a problem worthy of further study.

Research on this question can look back to 40s of 20^{th} century when the information theory was born. Shannon defined information as the decreasing of uncertainty. He also successfully solved the problem of information measuring and set up an inner link between uncertainty and information. By using entropy, a noun from statistical mechanics, which measures the disorder degree of system, uncertainty can be reflected by measuring amount of information or entropy in a future consumption system. It means, the more information or entropy is in a future consumption system, the more the degree of uncertainty is; on the contrary, the less information or entropy is in a future consumption, the less the degree of uncertainty is.

Assuming that a consumption system is influenced by several factors, accordingly there are S states $\{X_1, X_2, ..., X_S\}$, which constitute S-system. Considering items p_S, $\{P_1, P_2, ..., P_S\}$, $\forall i, P_i \neq 0$, $\sum_{i=1}^{S} P_i = 1$ having the same meanings with the function(3), through the Shannon Function in information theory, we can calculate Q, the uncertainty of S-system:

$$Q = -K \sum_{i=1}^{S} P_i \log P_i, \tag{29}$$

K is a positive constant.

A stable ideal consumption system is the one whose influential factors are certain. When consumers can foresee future states, uncertain degree is minimum, namely, $Q \approx 0$. But when consumers cannot foresee future in a system that is not stable, uncertain degree can reach the maximum ($Q \approx K \log S$). For any other situation of the consumption system, Q varies from the minimum to the maximum. If we want to reduce uncertainty, we may reduce our future states.

When paid earlier attention to uncertainty in 1921, L. R. Knight indistinctly admitted relation between uncertainty and future information, thought that information collection is a positive means to deal with uncertainty, and information is a negative metric of uncertainty. Uncertainty in a market will reduce while information quantity increases. So reducing uncertainty means decreasing economic cost. The founder of modern reasoning statistics, R. A. Fisher initiated his information quantity to measure uncertainty. From the point that random variable contains total parameter information, he thought that when uncertainty of variable increased, the discrete degree increased, information quantity of unknown parameter to the distribution decreased at the same time.

Nobel Economics Prize gainer Arrow once incisively pointed that, so-called information is the any observation result under changing probability efficiently according to conditional probability principle. While modifying the general balanced theory, L. R. Knight led into information factors and make them as useful means to decrease uncertainty. Actually, consumers make most decisions under an uncertain condition because of a defective environment they are facing. The increasing of future information only means a decreasing of uncertainty to some extent.

5 Enlightenments on policy

Uncertainty has become an important part in the economic theories on consumption. Generally speaking, only can macroscopic control policy and efficient interference through the governments weaken its influence in a consumptive system, and ensure stability and increase of domestic consumption.

Government should pay full attention to reducing uncertainty when making macro policy, for it has positive effect on consumption. From the analysis above, we can enter into reducing uncertainty through several ways as below:

(1) Unreasonable distributional policy which makes income-discrepancy larger, will lead to an increasing of uncertain degree. Thus, it has notable negative effects on consumption behavior. Lessening this discrepancy originated from allocation policy will reduce the possible existed future states S of consumers, and also reduce the maximum uncertainty, $Q \approx K \log S$.

(2) Increasing the transparency of policy, which may make consumers gain more information about their future income and expenditure, helps reducing consumers' uncertain degree about future states.

(3) Increasing the stability and continuity of policy, which makes a consumption system more stable, helps lowering consumers' judgment upon uncertain degree for future states.

(4) Enhancing system construction helps reducing the uncertainty of social uncertainty.

From the viewpoints of new-system economics, our ability to understand the environment is limited, which causes different individuals reacting differently to environment. System is what can reduce uncertainty of environment through setting up series of rules. Our rationality is too limited compared to the environmental uncertainty and complexity. So establishing and optimizing system, as well as improving system structure helps to reduce uncertainty, and makes up our limited rationality which brought difficulties in making choices.

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