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A model and its application for uncertainly group decision making*

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Abstract. To better solve the complicated problem of multi-attribute group decision making (MAGDM), we proposed a systemic approach by extending TOPSIS method. Linguistic assess terms of each group member were translated into the matrix of fuzzy triangular numbers. By using the incomplete preference information, multi-steps interactive procedure was applied for evaluating, selecting, ranking and approximating group ideal point step by step. In addition, the parameters of group satisfaction degree were designed to weigh group consensus and control the procedure; thus, the order of alternatives were given out conveniently. Therefore, the cumbersome for aggregating and computing group preference data could be avoid. The distance from each alternative to group Fuzzy Ideal Solution (FIS) was calculated to determine the ranking order of all alternatives. Our approach is useful in solving MAGDM problem with less structuralized and incomplete information. A MAGDM example was tested to demonstrate the utility of our method.

The result indicated that a interactive group decision support system of realistic methodological frame can be build and applied in uncertainly distributed remote decision making environment with more than one decision maker.

Keywords: multi-attribute group decision-making, fuzzy decision making, interactive group decision support system, TOPSIS method

1 Introduction

The increasing complexity of the socio-economic environment makes more difficulty for single decision maker(DM) to consider all relevant aspects of problem. Therefore, group members should be employed in decision making. A group decision making process is usually to be the process of reducing different individual preferences among objects in a given set to a single collective preference, or group preference. This will introduce a great deal of complexity into the analysis due to the incomplete information provided[7]. For example, a DM can not provide exact estimations of attribute weights or not to specify the preference in detail. The reasons for a DM provides only incomplete information are the follows: (1) a decision usually be made under time pressures and lack of data; (2) many of the attributes are intangible or non-monetary because they reflect social and environmental impacts; (3) a DM has limited attention and information processing capabilities[19−20].

Therefore, human judgments are vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numerical values. A more realistic approach could be to use linguistic variables to model human judgments. Other than the parameter value information (weights of attributes, utility value and preference of alternatives) is precisely or numerically assessed by group members in the traditional research of

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MAGDM, the real information provided by them are uncertain or incomplete information. With it a selection is not generally made in a single step and some additional information is required to get a final selection. From this point of view, an interactive procedure is required.

In recent years, the subject has received a great deal of attention from researchers in many disciplines[3,12,17,20]. Salo[16] develops an interactive approach for the aggregation of group members' preference judgments in the context of an evolving value representation; Soung[19,20] studied an interactive procedure for MAGDM problems with range-based preference information... Although each approach has its own characteristics, there may be some drawbacks that make itself difficult to apply to real world problem. Only a few studies have employed synthetic uncertain model and can be realized with computer information technology in group decision making environment.

In our research linguistic variables are used to capture fuzziness in decision information by pairwise comparison to of every attributes of alternatives are translated into triangular fuzzy numbers. Using fuzzy Analytic Hierarchy Process (AHP) procedure, the assessment information is processed into the individual normalized judgment matrix $\widetilde{z}_i, i = 1, 2, ..., m$. Extended Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)[5] is then applied to solve the problem in the uncertain environment. From it, we obtain the individual ideal solution vectors $\widetilde{M_i^+}, \widetilde{M_i^-}, i = 1, 2, ..., m$, then weighed based on the individual important weight vector w. A group positive ideal solution vector (GPIS) and a group negative ideal solution vector(GNIS) $\widetilde{M_G^+}, \widetilde{M_G^-}$ are generated , the best compromise alternative is defined as the one that has the shortest distance to GPIS and the farthest from GNIS. The approach degree of each alternative to group ideal solution (GIS) can be calculated to determine the approach degree and ranking order of every alternatives. The higher value of the approach degree for an alternative indicates that the alternative is closer to GIS. Group consensus level parameters can also be defined to conform whether the decision making procedure will be finished. All above, we designed and realized the special interactive procedure based on group consistency indices to obtain the group satisfaction ranking order alternatives according to each group member's linguistic assess term.

In this paper, the basic idea is as follows: (1) decision making problem is uncertain, each group member gives only incomplete information; (2) decision maker want to compare his/her information to other group members for a consensus result. Based on them, we suggested an interactive procedure with the incomplete information to get a final decision. In this procedure large amount of information need to communicate and process among group members so a support system of the interactive group decision-making should be developed.

2 Problem statement and incomplete information process

In this section, we describe the following uncertain multi-attribute group decision making problem: suppose there exist N possible alternatives $(A_1, A_2, ..., A_n)$ from which M decision makers $(P_1, P_2, ..., P_m)$ have to choose on the basis of P attributes $(C_1, C_2, ..., C_p)$, the rating of preference pairwise comparison to every attribute of alternatives given by decision makers are linguistic assess terms. Hence, the MAGDM models are characterized by the following components:

$A = a_{i=1,N}$: a finite set of N possible alternatives;
$C = c_{i=1,P}$: a set of indices of P attributes;
$P = p_{i=1,M}$: a set of indices of M group members that participate in the decision making procedure;
$S = s_{i=1,11}$: a set consisted of odd linguistic assess term ;
We also define the following expression:
$\widetilde{z}^i$: the fuzzy triangular numbers evaluation matrix of decision maker i;
$\widetilde{T}^i$: the normalized fuzzy evaluation matrix of decision maker i;
$\widetilde{W}$: the individual importance weight vector;
$\widetilde{M_i^+}, \widetilde{M_i^-}$: the positive and negative ideal solution vectors of decision maker i;
$\widetilde{M_G^+}, \widetilde{M_G^-}$: the positive and negative ideal solution vectors of group;
D_{ij}^+, D_{ij}^-: Euclidean distance between the fuzzy normalized evaluation value $\widetilde{t}_{ij}$ and group fuzzy ideal solution;

C: relatively approach degree matrix of group;
ϕ: the group satisfaction degree matrix.

2.1 Linguistic assessment term

In less-structured cases, group decision makers evaluate alternatives by pairwise comparison in all aspects. Usually, the results of evaluation are give out by a uncertain linguistic variable. In here, the value of linguistic variable are linguistic terms. It is very useful in situations where decision problems are too complex or too ill-defined to be described properly by using conventional quantitative expressions. In this paper, the performance ratings of alternatives on qualitative attributes were expressed using a set S of linguistic variable. Here S is a ranking set consisted on odd linguistic assessment term as follow:

$s_1 =$ EP(Extremely Poor to), $s_2 =$ VSP(Very Strongly Poor to), $s_3 =$ SP(Strongly Poor to),
$s_4 =$ VP(Very Poor to), $s_5 =$ MP(Moderately Poor to), $s_6 =$ ET(Equally To),
$s_7 =$ MG(Moderately Good to), $s_8 =$ VG(Very Good to), $s_9 =$ SG(Strongly Good to),
$s_{10} =$ VSG(Very Strongly Good to), $s_{11} =$ EG(Extremely Good to).

Certainly, set S have characteristics as follow:

(1) in order: if $i \geq j$, then $s_i \succeq s_j$, symbol " $\succeq''$ means "be superior or equal to";

(2) invert operator "Neg" when $j = T - i$, then $Neg(s_i) = s_j$, here, $T - 1$ means the numbers of linguistic assessment term in set S.

(3) $\max$ and $\min$ operator: when $s_i \succeq s_j$, exist $MAX(s_i, s_j) = s_i; MIN(s_i, s_j) = s_j$.

So the linguistic assess terms value can be represented using positive triangular fuzzy numbers conveniently.

2.2 Fuzzy triangular data process

In order to process the linguistic assessment term, these term should be translated into fuzzy triangle numbers. For the sake of simplicity and without loss of generality, we assume that all fuzzy numbers are triangular fuzzy numbers throughout the paper. Let $\widetilde{d} = (l, m, u)$ be a triangular fuzzy number, where the membership function $u_{\widetilde{d}}$ of $\widetilde{d}$ is given by

$$u_{\widetilde{d}}(x) = \begin{cases} 0, & x \leq l \\ (x-l)/(m-l), & l < x \leq m \\ (x-u)/(m-u), & m < x \leq u \\ 0, & x > u \end{cases} . \tag{1}$$

In this formula, $X \in R$, here R is real area, m is a mode of $\widetilde{d}$, and l and u are the low and upper limits of $\widetilde{d}$, $l \leq m \leq u$; the bigger the $u - l$ is, the stronger the degree of fuzzy is.

Suppose decision maker d_k given pairwise comparison for every attributes matixs of alternative set A in linguistic assess term as designed linguistic assessment set S. It can be translated into triangular fuzzy numbers according to the Table 1:

Because the designed fuzzy triangular numbers have character:

$$z_{uij} \geq z_{mij} \geq z_{lij}; \quad z_{lij} + z_{uji} = z_{mij} + z_{mji} = z_{uij} + z_{lji} = 1, \quad \forall i, j \in N. \tag{2}$$

They are used to translate linguistic assessment term matrix $Z = (z_{ij})_{n \times n}$ into compromise matrix.

3 Extended TOPSIS method

TOPSIS is a ranking order method by calculating distance between the ideal solution and alternatives, here the positive and negative ideal solution are virtual alternative evaluation, and used to establish a reference point for evaluation these alternatives. Based on the individual normalized judgment matrix, obtain the individual ideal solution vectors, then weighing according weight vector W of group member's power and obtain the group ideal solution, Finally, the best compromise assessment alternative is defined as the one that has the shortest distance to GPIS and the farthest from GNIS and the ranking order can be obtained.

Table 1. Corresponding linguistic assessment term and triangle fuzzy data

numbers	character define	comparison i to j	triangular fuzzy number
s_1	EP	Extremely Poor to	(0.0,0.0,0.1)
s_2	VSP	Very Strongly poor to	(0.0,0.1,0.2)
s_3	SP	Strongly poor to	(0.1,0.2,0.3)
s_4	VP	Very poor to	(0.2,0.2,0.4)
s_5	MP	Moderately Poor to	(0.3,0.4,0.5)
s_6	ET	Equally to	(0.4,0.5,0.6)
s_7	MG	Moderately Good to	(0.5,0.6,0.7)
s_8	VG	Very Good to	(0.6,0.7,0.8)
s_9	SG	Strongly Good to	(0.7,0.8,0.9)
s_{10}	VSG	very Strongly Good to	(0.8,0.9,1.0)
s_{11}	EG	Extremely Good to	(0.9,1.0,1.0)

3.1 Preference matrix of triangular fuzzy number

In general, attributes can be classified into two types: benefit attributes and cost attributes. In the procedure of evaluating alternatives, the benefit and cost attributes are all normalized into benefit attributes automatically. Here decision maker i pairwise assess alternatives at attribute 1 p and these values of linguistic assessment term can be transformed into fuzzy triangular number evaluation matrix $\widetilde{Z}^i$ as follow:

$$\widetilde{Z}^i = \begin{bmatrix} \widetilde{z}^i_{11}(1), \widetilde{z}^i_{12}(1), ..., \widetilde{z}^i_{1n}(1) & \widetilde{z}^i_{11}(2), \widetilde{z}^i_{12}(2), ..., \widetilde{z}^i_{1n}(2) & \cdots & \widetilde{z}^i_{11}(p), \widetilde{z}^i_{12}(p), ..., \widetilde{z}^i_{1n}(p) \\ \widetilde{z}^i_{21}(1), \widetilde{z}^i_{22}(1), ..., \widetilde{z}^i_{2n}(1) & \widetilde{z}^i_{21}(2), \widetilde{z}^i_{22}(2), ..., \widetilde{z}^i_{2n}(2) & \cdots & \widetilde{z}^i_{21}(p), \widetilde{z}^i_{22}(p), ..., \widetilde{z}^i_{2n}(p) \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{z}^i_{n1}(1), \widetilde{z}^i_{n2}(1), ..., \widetilde{z}^i_{nn}(1) & \widetilde{z}^i_{n1}(2), \widetilde{z}^i_{n2}(2), ..., \widetilde{z}^i_{nn}(2) & \cdots & \widetilde{z}^i_{n1}(p), \widetilde{z}^i_{n2}(p), ..., \widetilde{z}^i_{nn}(p) \end{bmatrix},$$

$i = 1, 2, ..., m.$

Application fuzzy AHP method, we can translate the fuzzy triangular number evaluation matrix $\widetilde{Z}^i$ into normalized evaluation matrix $\widetilde{T}^i, i = 1, 2, ..., m$:

$$\widetilde{T}^i = \begin{bmatrix} \widetilde{t}^i_{11} & \widetilde{t}^i_{12} & \cdots & \widetilde{t}^i_{1p} \\ \widetilde{t}^i_{21} & \widetilde{t}^i_{22} & \cdots & \widetilde{t}^i_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{t}^i_{n1} & \widetilde{t}^i_{n2} & \cdots & \widetilde{t}^i_{np} \end{bmatrix}, \quad i = 1, ..., m.$$

3.2 Individual fuzzy ideal solution $\widetilde{m}_i^+$ and $\widetilde{m}_i^-$

According to TOPSIS method the positive ideal solution vector of decision maker i $\widetilde{M}_i^+$ can get by choosing the maximum element in every row of normalized decision matrix $\widetilde{T}^i, i = 1, 2, ..., m$:

$$\widetilde{M}_i^+ = (\widetilde{M}_{i1}^+, \widetilde{M}_{i2}^+, \ldots, \widetilde{M}_{ip}^+), \quad i = 1, 2, ..., m. \tag{3}$$

Similarity, the negative ideal solution vector of decision maker i $\widetilde{M}_i^-$ can get by choosing the minimum element in every row of normalized decision matrix $\widetilde{T}^i, i = 1, 2, ..., m$:

$$\widetilde{M}_i^- = (\widetilde{M}_{i1}^-, \widetilde{M}_{i2}^-, \ldots, \widetilde{M}_{ip}^-), \quad i = 1, 2, ..., m. \tag{4}$$

3.3 Group fuzzy ideal solution $\widetilde{m}_g^+$ and $\widetilde{m}_g^-$

To consider in difference important of decision makers, we design a vector $\widetilde{W} = (\widetilde{w}_1, \widetilde{w}_2, ..., \widetilde{w}_m)^T$, and weighed individual ideal vectors $\widetilde{M}_i^+, \widetilde{M}_i^-, i = 1, 2, ..., m$ and got the group fuzzy decision matrix $\widetilde{Q}^+, \widetilde{Q}^-$;

$$\widetilde{q}_{ij}^+ = \widetilde{w}_1 \odot \widetilde{M}^{+1} \oplus \widetilde{w}_2 \odot \widetilde{M}^{+2} \oplus \cdots \oplus \widetilde{w}_m \odot \widetilde{M}^{+m} = (q_{ij}^+(l), q_{ij}^+(m); q_{ij}^+(u)), \tag{5}$$

$$\widetilde{q_{ij}} = \widetilde{w}_1 \odot \widetilde{M}^{-1} \oplus \widetilde{w}_2 \odot \widetilde{M}^{-2} \oplus \cdots \oplus \widetilde{w}_m \odot \widetilde{M}^{-m} = (q_{ij}^-(l), q_{ij}^-(m); q_{ij}^-(u)). \quad (6)$$

After weighed fuzzy decision matrix has been determined, the fuzzy group positive ideal solution $\widetilde{M}_G^+ = (\widetilde{M}_{G1}^+, \widetilde{M}_{G2}^+, \ldots, \widetilde{M}_{Gp}^+)$ can be determined from the group fuzzy weighted decision matrix $\widetilde{Q}^+$; the group ideal solution reflecting the group's belief in the relative superiority of all alternatives attribute comparison pair-wisely. Here, $\widetilde{m}_{Gj} = \widetilde{\max}\{\widetilde{q}_{j1}, \widetilde{q}_{j2}, \ldots, \widetilde{q}_{jn}\}, j = 1, \ldots, p$ is attribute C_j's fuzzy maximum set corresponding with fuzzy weight decision matrix $\widetilde{Q}^+$. Similarity, the group fuzzy negative ideal solution $\widetilde{M}_G^- = (\widetilde{M}_{G1}^-, \widetilde{M}_{G2}^-, \ldots, \widetilde{M}_{Gp}^-)$ can be obtained from the group fuzzy weighed decision matrix $\widetilde{Q}^-$.

According to the square of the weighed Euclidean distance, the distance D_{ij}^+ between individual pair-wisely comparison evaluation value $\widetilde{t}_{ij}$ of alternatives and the fuzzy ideal solution $\widetilde{M}_G^+$ can be calculated as follow:

$$D_{ij}^+ = \sqrt{\sum_{k=1}^{P} [d(\widetilde{t}_{jkl}^i, \widetilde{M}_{Gkl}^+) + d(\widetilde{t}_{jku}^i, \widetilde{M}_{Gku}^+)]^2} \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., m. \quad (7)$$

The distance D_{ij}^- between fuzzy negative ideal solution $\widetilde{M}_G^-$ and individual pairwise comparison evaluation value $\widetilde{t}_{ij}$ of alternatives can be calculated as follow

$$D_{ij}^- = \sqrt{\sum_{k=1}^{P} [d(\widetilde{t}_{jkl}^i, \widetilde{M}_{Gkl}^-) + d(\widetilde{t}_{jku}^i, \widetilde{M}_{Gku}^-)]^2} \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., m. \quad (8)$$

3.4 Relatively approach level calculation

In order to measure group consensus, relatively approach degree decision maker i between evaluating value of alternative j and group ideal solution can be defined as:

$$C_{ij} = D_{ij}^- / (D_{ij}^+ + D_{ij}^-), \quad 0 \le C_{ij} \le 1, \quad i = 1, ..., n, \quad j = 1, ..., m. \quad (9)$$

Here, for ideal compromise alternative, C_{ij} is 1; if negative ideal solution, then C_{ij} near 0, generally, C_{ij} between 1 and 0; the nearer 1 the C_{ij}, the nearer optimization alternative the compromise alternative. Based on this, the group approach matrix is:

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{bmatrix}.$$

3.5 Interacting process judgment

Although we can rank order of all alternatives descending according to $C_j = \sum_{i=1}^{m} C_{ji}, j = 1, ..., n$ we should consider the group consensus problem; that is, in order to confirm the above result is group consensus result or have a higher group satisfaction. We defined two soft indictors to weight group approach degree or group satisfaction: First, satisfaction value $c_0 (e.g. C_0 = 0.5)$ as a limen and satisfaction function as follows:

$$\varphi_{i,j} = \begin{cases} 1 & \text{when } c_{ij} >= C_0 \\ 0 & \text{all others} \end{cases}. \quad (10)$$

Transferring the group approach matrix into the 0-1 group satisfaction matrix φ with the formula. It provides information on how close or far the group's ranking order is from satisfaction level. Based on it, group satisfaction indicator (GSI) can be defined as $GSI = (\sum_{i=1}^{n} (\sum_{j=1}^{m} \varphi_{i,j}))/(n \times m)$. This formula means that the group satisfaction indictor is a ratio of number of nonzero elements in the matrix to number of matrix elements.

Because the difference preference exists in all group members for difference alternatives and attributes, in order to promote group satisfaction level communicating among members and modifying evaluation information through interactive procedure is needed. In this research, all members interacted with the GDSS platform for more information and modified their preference adjustment accordingly. When group members interacted for consensus decision, the indicators provided by GSI were used to observe the tendency of changing group preference and control the interactive procedures through indicating how to adjust individual preference according to group consensus. We confirm that in a cooperative decision making procedure nonzero elements of matrix φ increased step by step and finally arrived a designed valve of group satisfaction degree indictor GSI0(e.g. GSI0=0.6), it means that the procedure can be finished with designed group satisfaction.

3.6 Ranking order of alternatives

In the case of satisfaction degree indictor $GSI \geq$ GSI0, the group ranking order of all alternatives can be obtained by $C_j = \sum_{i=1}^{m} C_{ji} \quad j = 1, 2, ..., n$ in a descending order. The ranking order of alternatives $A_j(i = 1, 2, ..., n)$ determined by approach degree value with the preference given by the decision makers and based on group consensus.

4 GDSS: support interactive decision making

To implement the above suggested procedure, we have proposed a framework of interactive procedure and developed a interactive group decision support system(IGDSS) suitable to the uncertain environment. With ASP-NET programme languages as web page develop tool, local or wider area network INTERNET as remote distributed processing environment, relational SQL-SERVICE as database management system and graphic user interactive(GUI)for reducing information process time and effort of repetitive interactive procedure, the group members could communicate expediently and make decision each other through the IGDSS platform.

4.1 WEB based communication platform

In our IGDSS system, WEB based on INTERNET support the distributed remote group members communication and helps group members provide their opinions freely by showing them evaluation matrix and group ideal solutions after each step. For the convenience of user's input, system provides user friendly graphic interfaces so user only press corresponding items or buttons. In the brower/server structure decision makers working at terminal B input their evaluation in linguistic assess terms, all information are collected to terminal S by the facilitator who process and public information to WEB page of terminal B and control whole procedure.

4.2 Interactive procedure

An overall interactive procedure is shown in Fig. 1. The primary goal of this procedure is to increase group satisfaction level and finally to obtain the group consensus solution alternative ranking order. The specific description of each step is given at the following subsections:

Step 1. Terminal S collects alternatives and confirm attributes outside system, selects group decision members. Then, uploads information about alternatives and group members into IGDSS and set initiation system parameters ;

Step 2. Terminal B decision makers comparison pairwise and input their preference value of each alternative on each attribute, terminal S collects these linguistic variables value of every group members and translates into fuzzy triangular numbers;

Step 3. With fuzzy AHP and extended TOPSIS method, terminal S calculates individual normolized decision matrixs, selects the positive and negative ideal solution vectors of every makers, then, weighs on possibly different importance of group members, and gets group's fuzzy weighed decision matrix;

Step 4. Terminal S selecting the group ideal solution vectors and calculating Euclidean distance between $\widetilde{t}_{ij}$ and the fuzzy group ideal solution vectors, then, relative approach degree matrix $C = (c_{ij})_{n\times m}$ on evaluating value of alternative and group ideal solution vectors;

Step 5. Terminal S use defined soft indictors: (1) satisfaction value c_0 as a limen to translate the group approach matrix into the 0-1 group satisfaction matrix φ; (2) group satisfaction indicator GSI0 calculates GSI and estimates whether the interactive procedure can be finished;

Step 6. When $GSI \leq GSI0$ means the group consensus is dissatisfaction, more interactive procedures are needed among group members for each attribute of every alternatives. This procedure is carried out until to get clear preference information for making the group consensus ranking order, echo individual assessment information about alternatives and ideal solutions in the WEB page of terminal B;

Step 7. In the case of group satisfaction degree indictor $GSI \leq$ GSI0, go to Step 2 for a new evaluation, in terminal B every group member read information provided on WEB page and give their new preference assessments;

Step 8. In the case of satisfaction degree indictor $GSI \geq$ GSI0 the group decision procedure should be finished. Terminal S the group ranking order of all alternatives can be obtained by descending order. Facilitator run concluding program to save evaluation value, output results and end whole interactive procedure.

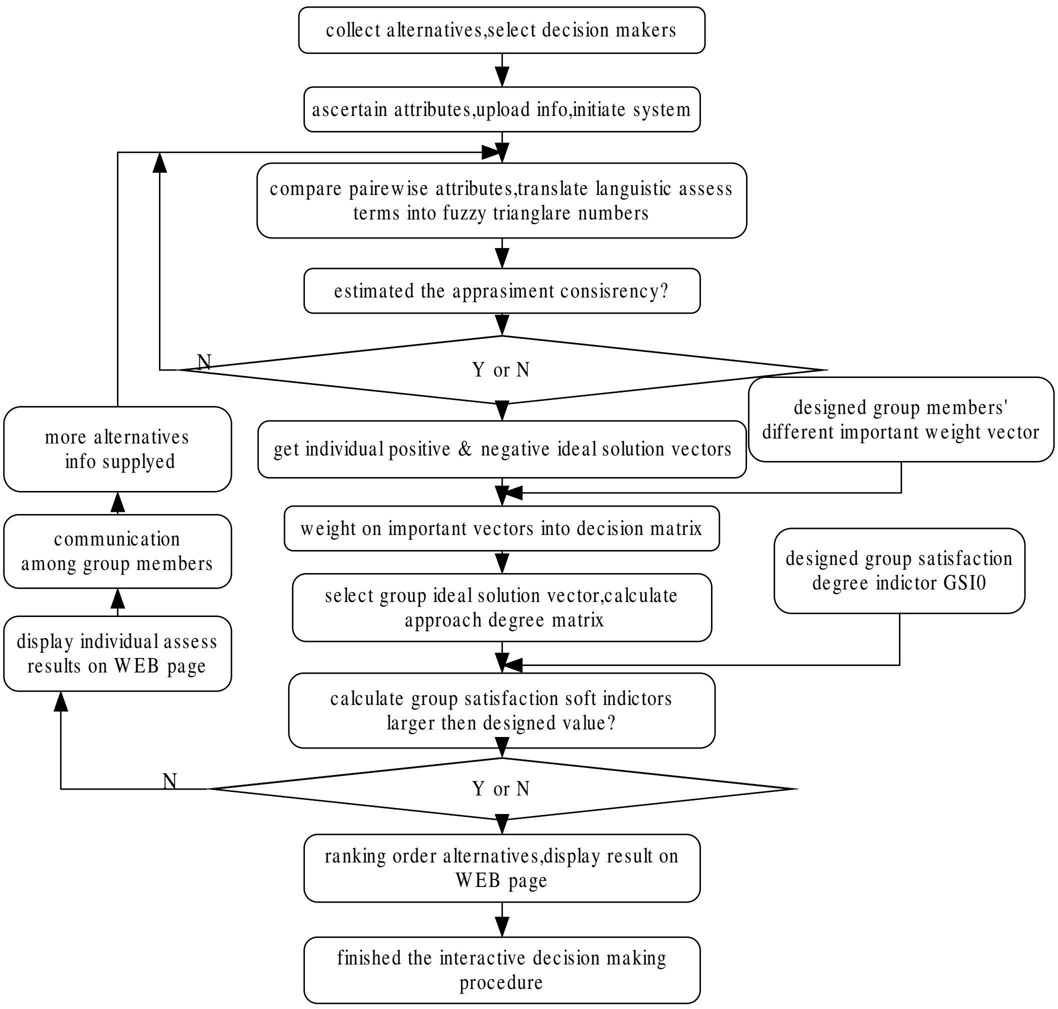

Fig. 1. The interactive procedure flowchart

The validation of this method has been proved and the responses of users are summarized as follows. (1) It is easy to use and has reduced the burden of group members providing incomplete preference information. (2) The gap between different judgments is easily reduced to reach a group's consensus with the method.

(3) By involving in the interactive process, group members will reach their final agreement more easier. This promote the level of their satisfaction on the decision making process and prove the utility of our suggested interactive procedure.

4.3 A numerical example

An numerical example of assessment selection problem from a scientific and technology institution was investigated in our research. Assume there are five institutional candidates $(A_1, A_2, A_3, A_4, A_5)$ that were evaluated by four experts (P_1, P_2, P_3, P_4). The four experts all agree to set following four objectives for the evaluating procedure; including (1) creditability (credit and reputation standing in their peer institution); (2) $R\&D$ Performance (performance in research and development); (3)potentiality(potential in science research and technology development); (4) self-improving capacity (potentials for advancement by themselves at aspects of marketing and management); the preference of experts to these qualitative attributes are expressed using linguistic variables. For the first expert P_1, the data and ratings of comparison pair-wise of all alternatives on every attribute are displayed in Table 2 The corresponding relations between linguistic variables

Table 2. The pairwise comparison result in linguistic term of expertp_1

creditability	$R\&D$ Performance	potentiality	self-improving capacity
ET, MG, ET, MG, ET	ET, VG, SG, VP, MG	ET, EG, MG, SG, EG	ET, SP, VSP, MG, VSG
MP, ET, MP, ET, MP	VP, ET, VG, SP, MP	EP, ET, VSP, SP, VP	SG, ET, MP, SG, EG
ET, MG, ET, ET, MP	SP, VP, ET, VSP, SP	MP, VSG, ET, SG, VSG	VSG, MG, ET, VSG, EG
MP, ET, ET, ET, MP	VG, SG, VSG, ET, VG	SP, SG, SP, ET, MG	MP, SP, VSP, ET, VSG
ET, MG, MG, MG, ET	MP, MG, SG, VP, ET	EP, VG, VSP, MP, ET	VSP, EP, EP, VSP, ET

and positive triangular fuzzy numbers list in Table 1. Running the fuzzy AHP model, the normalized decision matrix $\widetilde{T}^1$ of the expert P_1 can be obtain as follow, according to Tables 1 and 2.

$$\widetilde{T}^1 = \begin{bmatrix} \widetilde{t}^1_{11} & \widetilde{t}^1_{12} & \cdots & \widetilde{t}^1_{14} \\ \widetilde{t}^1_{21} & \widetilde{t}^1_{22} & \cdots & \widetilde{t}^1_{24} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{t}^1_{51} & \widetilde{t}^1_{52} & \cdots & \widetilde{t}^1_{54} \end{bmatrix}$$

$$= \begin{bmatrix} (0.147, 0.216, 0.320) & (0.160, 0.232, 0.340) & (0.230, 0.312, 0.412) & (0.122, 0.184, 0.275) \\ (0.113, 0.176, 0.270) & (0.107, 0.168, 0.260) & (0.047, 0.088, 0.157) & (0.203, 0.280, 0.382) \\ (0.133, 0.200, 0.300) & (0.053, 0.104, 0.180) & (0.203, 0.280, 0.392) & (0.230, 0.312, 0.422) \\ (0.120, 0.184, 0.280) & (0.207, 0.288, 0.410) & (0.122, 0.184, 0.275) & (0.108, 0.168, 0.255) \\ (0.153, 0.224, 0.330) & (0.140, 0.208, 0.310) & (0..88, 0.136, 0.216) & (0.027, 0.056, 0.118) \end{bmatrix}.$$

We use the fuzzy TOPSIS method to obtain the fuzzy ideal solution of individual P_1:

$$\widetilde{M}_1^+ = [(0.113, 0.176, 0.270)\ (0.053, 0.104, 0.180)\ (0.047, 0.088, 0.157)\ (0.027, 0.056, 0.118)],$$

$$\widetilde{M}_1^- = [(0.153, 0.224, 0.330)\ (0.207, 0.288, 0.410)\ (0.230, 0.312, 0.412)\ (0.230, 0.312, 0.422)].$$

Similarly,the normalized decision matrices and the fuzzy ideal solution of individual of the experts P_2, P_3, P_4 and P_5 can be obtained, respectively (data not shown).

Then to weigh with designed weighed vector W, (here, for the sake of simplicity and without loss of generality, assume weight vector as a accurate data $w_1 = 0.30; w_2 = 0.23; w_3 = 0.22; w_4 = 0.25$), then the group ideal solution vectors .

The square of the weighed Euclidean distance between pairwise comparison evaluation of alternatives and the fuzzy ideal solution are calculated, then relatively approach degree matrix C, assume satisfaction value $c_0 = 0.5$ as a limen and translate matrix C into matrix ϕ:

$$\phi = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

In case of group satisfaction degree limen $GSI0 = 0.6$, the $GSI = 10/20 = 0.5$ means the group consensus level GSI is not good enough, so that more interactive procedure is needed. After group members communicated with each other via GDSS platform for more information regarding group's tendency, and re-adjusted their preference on evaluation of alternatives , the matrix ϕ became:

$$\phi = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Now, the parameter $GSI = 12/20 = 0.6 \geq GSI0$ means that the group consensus level has achieved the designed value so the interactive procedure can be finished. The ranking order of the five alternatives by expert group is generated as follows: $X_2 \succ X_5 \succ X_3 \succ X_4 \succ X_1$.

5 Conclusions and further research

In this paper we focus on uncertain multi-attribute group decision-making and support system development method. There are two aspects of uncertain in this situation: (1) the preference information of experts is incomplete or fuzzy; (2) the evaluation selection action of group members is also unpredicted. Difficulty introduced by these need be solved through development a synthetic method, all works of our research bring four main achievements:

Firstly, because most multi-attribute decision making problems include qualitative attributes that are often assessed using incomplete information fuzzy set theory is well applicable to dealing with such decision problems. The method which we suggest to process qualitative linguistic assessment term into fuzzy triangular numbers is suitable to the uncertain environment;

Secondly, we extend AHP and TOPSIS to fuzzy situation. The classical TOPSIS method is further developed to solve multi-attribute group decision making problems in fuzzy environments. A fuzzy TOPSIS model is constructed to rank alternatives using the pair-wisely comparison;

Thirdly, we design a set of soft indictors to judgment whether or not the result is consensus and satisfaction decision. Whether or not a group as whole satisfy with the result of evaluation ranking order, we provide formula $GSI \geq GSI0$ as a criterion to finish interactive procedure and gain final reasonable result;

Fourthly, we developed a simulation model IGDSS which can support communicating among decision makers to generate consistent and reliable ranking order of alternatives in uncertain MAGDK situation. Validation of the IGDSS method has done by an example of evaluation problem in science research organization;

All in all, the research is expected to be applicable to decision problems in many areas, especially in situations where multiple decision makers are involved and the evaluation information are incomplete.

Although our research, especially in developing the application of IGDSS is currently at beginning stage we are still able to see the attractions and potentials for its implementation. We present this paper with the hope that it will attract suggestions and comments from other researchers and system developers, this will help us to improve our framework and increase the possibilities for successful use in uncertain group decision-making situations. Future research efforts could result in the definition of better indicators. Such efforts could involve (1) the use of other methods for measuring the level of agreement between each pair of group members; and (2) the definition of other group and individual consensus indicators.

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