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The integration of valued outranking relations in ELECTRE methods for ranking problem *

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Abstract. The ELECTRE family of decision aid methods are well-known approaches in the field of multiple criteria decision aiding (MCDA). Different ELECTRE methods can be used for choice problem, ranking problem and sorting problem respectively. These methods are characterized by outranking relations and exploitation procedures. Different definitions of the outranking relations result in the difference of the exploitation procedures. It makes the DM difficult to choose a appropriate method to deal with the decision problem. On the other hand, the essence of the original ELECTRE is to evaluate the outranking relation of two alternatives from two different aspects, i.e. the concordance and the discordance. So, it is expected that the two aspects are independent. However, in classic ELECTRE methods, the correlation between two parameters still exist. More importantly, the uncertainty and imprecision are always exist in real-life situations. But the definition of the outranking relation in ELECTRE-I and ELECTRE-II methods can't handle the phenomena very well. The ELECTRE-III method uses the concept of pseudo-criterion to handle these uncertain or imprecise phenomena. But the definition of the outranking relation is still incomplete.

This paper aims at integrating the valued outranking relation on the basis of ELECTRE-I, ELECTRE-II and ELECTRE-III methods for ranking problem. The new outranking relation incudes most information of these three methods and the three methods can be treated as three special situations of the new relation. By introducing the discordance index and the preponderance index as well as the non-linear form of the concordance index and the discordance index, the new outranking relation is more complete and more suitable to simulate the non-linear and uncertain real-life situations. An illustrative example has been given to prove the rationality of this method for ranking problem.

Keywords: discordance threshold, preponderance index, valued outranking relation, integration, ELECTRE method, ranking problem, MCDA

1 Introduction

In the MCDA problem, several binary relations are often constructed to represent the preference among pairs of alternatives. One of the most important binary relations called "outranking relation" is constructed by outranking methods which are developed by the European School of MCDA[13, 20]. The ELECTRE methods are the first representatives of this family. Each ELECTRE method has been proposed to deal with a class of specific problems. The original version of ELECTRE has been presented for the choice of a acyclic graph[1]. But it is not well-suited to the new problem that oriented towards building of a ranking. Hence, the ELECTRE-II method has been developed[15, 16]. The ELECTRE-III method has been presented for the imprecise and even

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ill-determined performances of the alternatives. It works with three thresholds, i.e. indifference, preference and veto threshold. It also introduces a fuzzy outranking relation instead of a preference model containing only two crisp outranking relations[10]. ELECTRE-IV method which provides ranking of alternatives, has been designed for the case in which it is particularly difficult to indicate the relative importance of each criterion[19]. Two other ELECTRE methods have been designed so as to evaluate the intrinsic value each alternative by assigning it to predefined categories, i.e. sorting problem[9, 24]. A comprehensive treatment of ELECTRE methods can be found in the books by B. Roy and D. Bouyssou[18] and Ph. Vincke[23].

All of those ELECTRE methods comprise two main procedures: building the outranking relation[12] and exploiting it with regard to the chosen statement of the problem[22]. The construction of one or several outranking relation(s) aims at comparing in a comprehensive way each pair of actions. It assumes that one alternative dominate another one if there are sufficient advantages and no significant disadvantages. The exploitation procedure is used to elaborate recommendations from the results obtained in the first phase. The nature of the recommendations depends on the problem. Hence, each method is characterized by its construction of outranking relation and its exploitation procedures.

As stated by B. Roy, there are three basic MCDA problems [14]: the choice problem that aims at selecting a subset of potential alternatives, as small as possible, containing the "satisfactory" actions; the ranking problem that aims at ranking the alternatives by decreasing order of preference and the sorting problem that corresponds to the assignment of each alternative into pre-defined categories. The ELECTRE-I, ELECTRE-IS methods are mainly designed to deal with the choice problem. The ELECTRE-II, ELECTRE-III and ELECTRE-IV methods mainly aim at the ranking problem. ELECTRE-TRI aims at the sorting problem[4].

In these ELECTRE methods, each outranking relation has been born of difficulties encountered with a specific, concrete problem. All of these relations are defined separate and only used in one ELECTRE method. These methods make the DM difficult to choose an appropriate method when they are confronted with the ranking problem. Actually, the outranking relations defined in those methods have a lot in common. They can be defined in an integrative way. On the other hand, the essence of the original ELECTRE is to define the outranking relation from the concordance and discordance aspects. It is expected that these two aspects are independent. But sometimes the correlation still exists. Moreover, all the outranking relations in these three methods are defined in linear form. Actually, the non-linear forms are extensively exist.

In addition, alternatives are usually evaluated from different points of view which correspond to criteria. However, in real-life situations, evaluations are neither certain nor precise. There are three main sources of this uncertainty[17]: (1)imprecision caused by the difficulty of determining the value of action on particular criteria; (2)indetermination, since the method of evaluation results from a relatively arbitrary choice among several possible definitions; (3)uncertainty, since the values involved vary over time and space. So, the fuzzy logic should be taken into account when ranking the alternatives. The ELECTRE-I method construct the outranking relation by combining the crisp concordance index and discordance index. It can't deal with the uncertainty or imprecision at all. In ELECTRE-II method, the fuzzy logic is somewhat reflected, but mainly in the discordance index. The ELECTRE-III method uses the concept of pseudo-criterion to handle these uncertain or imprecise phenomena[11, 18]. But the definition of the outranking relation is deficient. Especially, the correlation between the concordance index and discordance index still exists. Therefore, the existent outranking relation can't meet the demands of the DM. All of these problem mentioned above call out the new definition of the outranking relation.

In this article, we propose an integrative outranking relation for the ranking problem considering the problems mentioned above. It comes from ELECTRE-I, ELECTRE-II and ELECTRE-III methods and includes nearly all information of these three methods, but it is more comprehensive than these ones mentioned above. These three methods can be considered as three special situations of the comprehensive ELECTRE method by changing some parameters of the new method. By the aggregating and integrating, the new ELECTRE method becomes more flexible for the ranking problem, and more adaptive to the uncertain situation. And the correlation between the concordance index and discordance index is eliminated by introducing a new parameter. What should be pointed out is that the ELECTRE-I method was originally designed for choice problem, but it can be used to deal with the ranking problem by changing some steps of the constructing procedures and the exploitation procedures. The ELECTRE-IV method aims at dealing with ranking problem, but it's not concerned in our discussion because of the lack of assigning an importance coefficient to the relative criterion as those three methods.

The paper is organized as follows. The next section provides a brief reminder on the basic definitions of the concordance and discordance conditions as well as the outranking relation. In section 3, the new integrative outranking relation is proposed. Based on it, the computational aspects of the new integrative ELECTRE method are presented in section 4 and an illustrate example has been given to expatiate on this method in section 5. In section 6, the discussion shows the relation between the new outranking relation and those three ELECTRE methods. Section 7 draws some conclusions and issues for further research.

2 Background aspects

Let us consider a decision situation involving a finite set of alternatives $A = \{a_1, a_2, \cdots, a_m\}$ evaluated on n criteria $g_1, g_2, \cdots, g_n$. Let $F = \{1, 2, \cdots, n\}$ denote the set of all criteria. Without loss of generality, we will assume the preferences are increasing with value on g_j, i.e., the greater $g_j(a)$ implies the better a. The decision matrix is given as follows:

$$D = \begin{bmatrix} g_1(1) & g_2(1) & \cdots & g_n(1) \\ g_1(2) & g_2(2) & \cdots & g_n(2) \\ \vdots & \vdots & \ddots & \vdots \\ g_1(m) & g_2(m) & \cdots & g_n(m) \end{bmatrix}.$$

The outranking relation of $a \to b$ (denoted as aSb and means "a is at least as good as b" or "a is not worse than b") says that even though two alternatives a and b do not dominate each other mathematically, the decision maker accepts the risk of regarding a as almost surely better than b[5, 8]. The construction of an outranking relation S is based on the comparing of all the pairs of alternatives $(a, b) \in A^2$. The comparison is grounded on the evaluation vectors of both alternatives a and b, i.e. $(g_1(a), g_2(a), \cdots, g_n(a))$ and $(g_1(b), g_2(b), \cdots, g_n(b))$, and on additional information concerning the DM's preference. To validate a statement aSb, some conditions should be verified. The most basic and important conditions are concordance and discordance conditions. They have been defined from two procedures: the concordance or discordance index for single criterion and the overall concordance or discordance relation.

A criterion g_j is said to be concordant with the assertion aSb if a is at least as good as b with respect to criterion g_j. A criterion g_j is said to oppose a veto to the assertion aSb if the difference of evaluation $g_j(b) - g_j(a)$ is incompatible with the assertion aSb, whatever the evaluation on the other criteria. The concordance index $S_j(a, b)$ or discordance index $d_j(a, b)$ for single criterion can be defined in crisp way or fuzzy way. The typical crisp indices can be found in ELECTRE-I method[1, 6]:

$$S_j(a,b) = \begin{cases} 1, & g_j(a) \geqslant g_j(b) \\ 0, & g_j(a) < g_j(b) \end{cases},$$

$$d_j(a,b) = \begin{cases} 0, & g_j(b) - g_j(a) < v_j(g_j(a)) \\ 1, & g_j(b) - g_j(a) \geqslant v_j(g_j(a)) \end{cases},$$

where, $v_j(g_j)$ is the veto threshold. The typical fuzzy indices can be found in ELECTRE-III method. In this method, three thresholds have been specified to construct the fuzzy concordance index and discordance index: the indifference threshold $q_j(g_j)$, the preference threshold $p_j(g_j)$ and the veto threshold $v_j(g_j)$ $(0 < q_j(g_j) \leqslant p_j(g_j) \leqslant v_j(g_j))$[10]. The interpretation for these thresholds under the criterion g_j are given as the following:

$g_j(a) \geqslant g_j(b) + q_j(g_j(a))$ indicates a is at least as good as b;
$g_j(a) \geqslant g_j(b) + p_j(g_j(a))$ indicates a is strctly preferred to b;
$g_j(a) \geqslant g_j(b) + v_j(g_j(a))$ indicates a is largely than b.

Hence, the fuzzy concordance index and discordance for the single criterion are respectively defined as:

$$S_j(a,b)=\begin{cases}1, & g_j(a)+q_j(g_j(a))\geqslant g_j(b)\\ \dfrac{g_j(b)-(g_j(a)+p_j(g_j(a))}{q_j(g_j(a))-p_j(g_j(a))}, & g_j(a)+q_j(g_j(a))<g_j(b)<g_j(a)+p_j(g_j(a))\\ 0, & g_j(b)\geqslant g_j(a)+p_j(g_j(a))\end{cases},$$

$$d_j(a,b)=\begin{cases}0, & g_j(a)+p_j(g_j(a))\geqslant g_j(b)\\ \dfrac{g_j(b)-(g_j(a)+p_j(g_j(a))}{v_j(g_j(a))-p_j(g_j(a))}, & g_j(a)+p_j(g_j(a))<g_j(b)<g_j(a)+v_j(g_j(a))\\ 1, & g_j(b)\geqslant g_j(a)+v_j(g_j(a))\end{cases}.$$

The overall concordance relation $C(a,b)$ is grounded on the index $S_j(j=1,2,\cdots,n)$ and fulfilled for the assertion aSb iff the subset of criteria concordant with aSb is "sufficiently" large. When computing this majority level, each criterion g_j has a weight $w_j\geqslant 0$ representing its voting power. Without any loss of generality, we will consider:

$$\sum_{j=1}^{n} w_j=1.$$

Hence, the overall concordance relation is obtained:

$$C(a,b)=\sum_{j=1}^{n} w_j S_j(a,b).$$

The overall non-discordance relation $ND(a,b)$ is grounded on d_j and fulfilled for the assertion aSb iff no criterion opposes a veto to the assertion aSb. It is defined in different way in different ELECTRE methods:

$$ND(a,b)=\prod_{j\in F}(1-d_j(a,b)) \quad \text{(in ELECTRE-I)},$$

$$ND(a,b)=\prod_{j\in\overline{F}:\{j\in F|d_j(a,b)>C(a,b)\}}\frac{1-d_j(a,b)}{1-C(a,b)} \quad \text{(in ELECTRE-III)}.$$

Hence, the classic valued outranking relation grounded on the concordance and non-discordance relation has been defined as:

$$S(a,b)=C(a,b)ND(a,b).$$

The outranking relation can be crisp or fuzzy form. Given two alternatives a and b, a crisp outranking relation $S(a,b)$ indicates a certain relation of a and b. $S(a,b)$ can be valued as 1 or 0. $S(a,b)=1$ implies a certain outranking of a over b. That means the analyst has enough reasons to admit that in the eyes of the DM, a is at least as good as b. Hence, a is indifferent from or preferred to b. $S(a,b)=0$ expresses "a does not outrank b", and it implies that the arguments in favor of the proposition "a is at least as good as b" are judged insufficient and that there exist arguments in favor of "b is at least as good as a". Hence, b is preferred or incomparable to a[9].

A fuzzy outranking relation $S(a,b)$ indicates the degree of outranking associated with each pair of alternatives (a,b). It shows the credibility of preference existing of a over b and increase with the reliability of the outranking of a over b. Particularly, $S(a,b)$ is a nondecreasing function of $g_j(a),\forall j$ and a nonincreasing function of $g_j(b),\forall j$. $S(a,b)=1$ implies a certain outranking of a over b, while $S(a,b)=0$ indicates a certain nonoutranking of b by a or the total absence of arguments in favor an outranking. More properties of the fuzzy relation $S(a,b)$ can be found in [9, 21, 25]. This class of relation is typically used in ELECTRE-III method. During the Exploitation phase, this binary valued relation may be transformed into a crisp relation by introducing a cut threshold $(\lambda-s(\lambda))$[10, 23]:

$$aSb \quad \text{iff} \quad S(a,b)\geqslant\lambda-s(\lambda).$$

3 Integrative outranking relation

The integrative valued outranking relation is originally defined grounded on the three ELECTRE methods mentioned above. Although these methods are characterized by their constructions of the outranking relations and the exploitation procedures given following, they have a great deal in common especially with respect to the ranking problem. Actually, these analogical outranking relations can not only be used in one method but also can be used in the other ELECTRE methods, i.e., they can be uniformed. This method is designed to provide a integrative and uniform form in ELECTRE for ranking problem.

3.1 Partial index for single criterion

Three thresholds have been proposed in classic ELECTRE-III method: the indifference threshold $q_j(g_j)$, the preference threshold $p_j(g_j)$ and the veto threshold $v_j(g_j)$. Two thresholds $q_j(g_j)$ and $p_j(g_j)$ are used to construct the concordance index S_j. Similarly, $p_j(g_j)$ and $v_j(g_j)$ are used to construct the discordance index d_j. The same threshold $p_j(g_j)$ is used in two indices. Hence, there are some relations existing in these two indices. A change on one index may affect the other, i.e., these two indices are correlative. For example, if $p_j(g_j)$ is changed smaller, the concordance index will be relaxed, and the discordance index will also be relaxed. Hence, the effect is magnified. Considering that the essence of the original ELECTRE method is to evaluate the outranking relation from two different aspects, the correlation between those ones should be as small as possible. A simple way is adding a threshold to make this two indices independent. It can be denoted by $u_j(g_j)$. This threshold can be smaller or larger than $p_j(g_j)$. But for the purpose of uniforming the overall non-discordance relation, we define the threshold $u_j(g_j)$ larger than $p_j(g_j)$. $u_j(g_j)$ represents the difference of evaluation $g_j(b) - g_j(a)$ above which the discordance condition starts to weaken concordance $C(a,b)$ in the definition of $ND(a,b)$ in ELECTRE-III method. So, we can call this threshold "discordance threshold", such that $p_j(g_j) \leqslant u_j(g_j) \leqslant v_j(g_j)$ [7]. In addition, this index mentioned in ELECTRE methods is linear. It is the most simple situation. Actually, we can consider it as linear form or non-linear form[21]. Hence, the concordance index and discordance index have been obtained:

$$S_j(a,b) = \begin{cases} 1, & g_j(a) + q_j(g_j(a)) \geqslant g_j(b) \\ \left(\dfrac{g_j(b) - (g_j(a) + p_j(g_j(a))}{q_j(g_j(a)) - p_j(g_j(a))} \right)^L, & g_j(a) + q_j(g_j(a)) < g_j(b) < g_j(a) + p_j(g_j(a)), L > 0 \\ 0, & g_j(b) \geqslant g_j(a) + p_j(g_j(a)) \end{cases},$$

$$d_j(a,b) = \begin{cases} 0, & g_j(a) + u_j(g_j(a)) \geqslant g_j(b) \\ \left(\dfrac{g_j(b) - (g_j(a) + u_j(g_j(a))}{v_j(g_j(a)) - u_j(g_j(a))} \right)^L, & g_j(a) + u_j(g_j(a)) < g_j(b) < g_j(a) + v_j(g_j(a)), L > 0 \\ 1, & g_j(b) \geqslant g_j(a) + v_j(g_j(a)) \end{cases}.$$

According to that, the decrease of $S_j(a,b)$ and the increase of the discordance ($d_j(a,b)$) can be determined by linear interpolation (when $L = 1$) or any other form (when $L \neq 1$).

3.2 Overall concordance and discordance conditions

As mentioned above, the overall concordance relation $C(a,b)$ is considered as the most important concordance condition in classical ELECTRE methods. But still exists the situation that the evaluation may be different from that the DM thinks, whereas the condition above is fulfilled. The reason is that the satisfaction of the concordance condition may result not from the dominance on some criteria but from the indifference on those criteria to a great extent. Hence, another index to describe the dominance of the preference on the criteria of a pair of alternatives (a,b) should be introduced. It should reach a certain level. In classic ELECTRE methods, a similar idea has been proposed especially in ELECTRE-II method[3, 15, 16], but it hasn't attached importance. In this paper, a way to evaluate that has been proposed as:

$$C'(a,b) = \begin{cases} 1, & W^+ \geqslant W^- \\ 0, & W^+ < W^- \end{cases},$$

where, W^+ is the sum of the weight for which $g_j(a) \geqslant g_j(b) + p_j(g_j(a))$, and W^- is the sum of the weight for which $g_j(a) + p_j(g_j(a)) \leqslant g_j(b)$. This index can be defined as the "preponderance index". Hence, the concordance condition includes two parameters, i.e., the overall concordance relation $C(a,b)$ and the preponderance index $C'(a,b)$. The concordance condition is fulfilled if and only if $C(a,b)$ reaches a certain degree and $C'(a,b)$ equals to 1.

The overall discordance condition is described by the overall non-discordance relation $ND(a,b)$. It is defined as:

$$ND(a,b) = \prod_{j \in F}(1 - d_j(a,b)).$$

A smaller value of $ND(a,b)$ indicates a higher discordance.

3.3 Valued outranking relation

The concordance condition and discordance condition are then combined to give the fuzzy valued outranking relation $S(a,b)$ as:

$$S(a,b) = C(a,b)C'(a,b)ND(a,b).$$

4 Algorithm of the integrative ELECTRE

So far, based on the integrative outranking relation discussed above, the new ELECTRE method for ranking problem has come out. It comes from ELECTRE-I, ELECTRE-II and ELECTRE-III methods and includes most information of them. But, it's different from those ones. The information included in the integrative ELECTRE method is more abundant. And the parameters in this method is more flexible to be controlled. The algorithms of the integrative ELECTRE method can be presented as follows:

Step 1. Initialization. The DM must set the thresholds: indifference threshold $q_j(g_j)$, preference threshold $p_j(g_j)$, discordance threshold $u_j(g_j)$ and veto threshold $v_j(g_j)$ $(q_j(g_j) \leqslant p_j(g_j) \leqslant u_j(g_j) \leqslant v_j(g_j))$.

Step 2. Construct the concordance index and discordance index for each criterion. Note that, the concordance index and the discordance index can be builded in a linear form or a non-linear form. It depends on the DM and the problem which is discussed.

Step 3. Calculate the overall concordance condition and discordance condition. In this step, three parameters, i.e. $C(a,b)$,$C'(a,b)$ and $ND(a,b)$, should be calculated.

Step 4. Combine the concordance and discordance conditions to obtain the fuzzy outranking relation $S(a,b)$.

Step 5. Rank the alternatives by using the exploitation procedure. The exploitation procedures in ELECTRE-II and ELECTRE-III methods can be similarly used in this step. Firstly, a value $\lambda = \max\limits_{(a,b)\in F} S(a,b)$ is determined and only the arcs having "sufficiently close" to λ are considered, i.e. more precisely, those which have a value larger or equal to $\lambda - s(\lambda)$, where $s(\lambda)$ is a threshold to be determined. The latter yields a non-valued outranking relation for which the qualification $Q(a)$ of each action a can be computed, that is the number of actions which are outranked by a minus the number of actions which outrank a. The set of actions having the largest qualification will be called the first distillate D_1. If D_1 only contains one action, the previous procedure is started again in $A \setminus D_1$. Otherwise, the same procedure is applied inside D_1; if distillate D_2 which is thereby obtained is a singleton, the procedure is started again in $D_1 \setminus D_2$ except if the latter set is empty; otherwise, it is applied inside D_2, and so forth until D_1 is used up entirely, before starting with $A \setminus D_1$. This procedure, which is called a descending distillation chain, yields a first complete preorder r'. A second complete preorder r'' is obtained by an ascending distillation chain, in which the actions having the smallest qualification are first retained and ranked last. The final ranking r can be obtained, similarly as suggested by Roy and Bertier[15], by taking the average of r' and r'', i.e. $r = (r' + r'')/2$. The alternative which gets least average value is ranked first and the alternative having next value is ranked second and so on till all elements of the alternatives are ranked.

5 The illustrative example

The data used in the illustrative example are from the preference [2]. Six criteria have been used to evaluate the business value of a rule: Degree of change (DoC), Support (Sup), Confidence (Conf), Interest factor (IF), Expected monetary value (EMV) and Incremental monetary value (IMV). The decision matrix of the rule prioritization problem is given in Table 1 and the weights are given by the DM as:

$$W = (0.330, 0.165, 0.065, 0.044, 0.164, 0.232).$$

All the calculations below are based on the software MATLAB 7.0.

Table 1. Decision table

Alternatives	DoC	Sup	Conf	IF	EMV	IMV
Rule 1	1.734	0.003332	0.098	2.090	9800	5112.024
Rule 2	-0.016	0.002430	0.081	20.468	2430	2311.279
Rule 3	-0.032	0.002759	0.089	22.490	2670	2551.279
Rule 4	0.741	0.002720	0.080	20.215	2400	2281.279
Rule 5	-0.258	0.002752	0.086	1.002	15,394	27.790
Rule 6	0.741	0.004230	0.141	2.842	12,690	8224.247
Rule 7	0.741	0.004300	0.086	2.854	7740	5027.671

Step 1. Initialization. The thresholds of the six criteria are set by the DM (Table 2).

Table 2. Thresholds table

Thresholds	DoC	Sup	Conf	IF	EMV	IMV
$q_j(g_j)$	0.2	0.0002	0.002	3	1500	1000
$p_j(g_j)$	0.5	0.0007	0.02	6	3000	2000
$u_j(g_j)$	0.7	0.001	0.05	8	4000	5500
$v_j(g_j)$	1.5	0.002	0.07	20	10000	6500

Step 2. Construct the concordance index and discordance index for each criterion. Supposing that the concordance index for each criterion is linear ($L = 1$) and the discordance index for each criterion is non-linear ($L = 1/2$), we can obtain the concordance indices and discordance indices. For example, the concordance index on the third criterion "Conf" and discordance index on the fifth criterion "EMV" are given as:

$$S_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0.1667 & 1 & 0.6667 & 1 & 0.8333 & 0 & 0.8333 \\ 0.6111 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0.1111 & 1 & 0.6111 & 1 & 0.7778 & 0 & 0.7778 \\ 0.4444 & 1 & 0.9444 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0.4444 & 1 & 0.9444 & 1 & 1 & 0 & 1 \end{bmatrix},$$

$$d_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.5154 & 0 & 0 \\ 0.7494 & 0 & 0 & 0 & 1 & 1 & 0.4673 \\ 0.7223 & 0 & 0 & 0 & 1 & 1 & 0.4223 \\ 0.7528 & 0 & 0 & 0 & 1 & 1 & 0.4726 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7804 & 0.3979 & 0 \end{bmatrix}.$$

The other indices of the rest of the criteria can be given similarly.

Step 3. Calculate the concordance conditions C, C' and discordance condition ND. The matrix of concordance conditions for all the alternative pairs are given as follows (7×7):

$$C = \begin{bmatrix} 1 & 0.9560 & 0.9560 & 0.9560 & 0.8360 & 0.3860 & 0.8350 \\ 0.0548 & 1 & 0.9358 & 0.6403 & 0.7849 & 0.0440 & 0.0982 \\ 0.1256 & 1 & 1 & 0.6700 & 0.8360 & 0.0440 & 0.1090 \\ 0.0803 & 1 & 0.9747 & 1 & 0.8216 & 0.3740 & 0.4246 \\ 0.2765 & 0.6778 & 0.6918 & 0.3940 & 1 & 0.2080 & 0.2730 \\ 0.6700 & 0.9560 & 0.9560 & 0.9560 & 0.8684 & 1 & 1 \\ 0.5727 & 0.9560 & 0.9524 & 0.9560 & 0.8360 & 0.5390 & 1 \end{bmatrix},$$

$$C' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

The discordance condition are given as:

$$ND = \begin{bmatrix} 1 & 0.0700 & 0 & 0.0814 & 0.4846 & 1 & 1 \\ 0 & 1 & 1 & 0.7331 & 0 & 0 & 0.0263 \\ 0 & 1 & 1 & 0.6979 & 0 & 0 & 0.1066 \\ 0.0976 & 1 & 1 & 1 & 0 & 0 & 0.1257 \\ 0 & 0.0225 & 0 & 0.0129 & 1 & 0 & 0.1009 \\ 0.3948 & 0.1044 & 0.0148 & 0.1162 & 1 & 1 & 1 \\ 0.3948 & 0.1049 & 0.0153 & 0.1168 & 0.2196 & 0.3011 & 1 \end{bmatrix}.$$

Step 4. Obtain the fuzzy outranking relation S:

$$S = \begin{bmatrix} 1 & 0.0669 & 0 & 0.0778 & 0.4051 & 0 & 0.8350 \\ 0 & 1 & 0.9358 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.9747 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.2645 & 0.0998 & 0.0141 & 0.1111 & 0.8684 & 1 & 1 \\ 0 & 0.1003 & 0.0146 & 0.1117 & 0.1836 & 0 & 1 \end{bmatrix}.$$

Step 5. Rank the alternatives. Firstly, let $s(\lambda) = 0.9$, we can get $\lambda - s(\lambda) = 0.1$. The set of actions having the largest qualification is obtained, i.e $D1$ =Rule 6. Then, $D2 = \{$Rule 1, Rule 7 $\}$is obtained. The similar procedure is applied inside $D2$. From that, we can get that Rule 1 is preferred to Rule 7. Hence, through the descending distillation chain, the complete preorder r' of all alternatives is obtained. Fig. 1 shows the process of this distillation chain. The second complete preorder r'' is obtained similarly by an ascending distillation chain, see Fig. 2. The final ranking r is given in Table 3.

It is found from Table 3 that Rule 6 and Rule 1 get the first rank and Rule 5 gets the last rank. Comparing with the ranking result in [2], we can find the alternatives Rule 4 and Rule 3 change a lot. The order of Rule 4 ascends largely and is preferred to Rule 3. It can be found that almost all of the criteria values on Rule 3 are larger than that on Rule 4 except the first criterion "DoC". However, according to the thresholds defined in this paper, the pair of alternatives Rule 3 and Rule 4 are indifferent on most of these criteria. Moreover, Rule 4 is strictly preferred to Rule 3 in the first criterion "DoC" (i.e $g_1(4) \geqslant g_1(3) + p_1(g_1(4))$) and this criterion is the most important one of all the criteria. Hence, the alternative rule 4 is preferred to rule 3 at last. The alternative 5 gets the last rank mainly because the values on the most important criteria "DoC" and "IMV" are further worse than the others and the values in other criteria are also not very good. The other alternatives can be analyzed similarly.

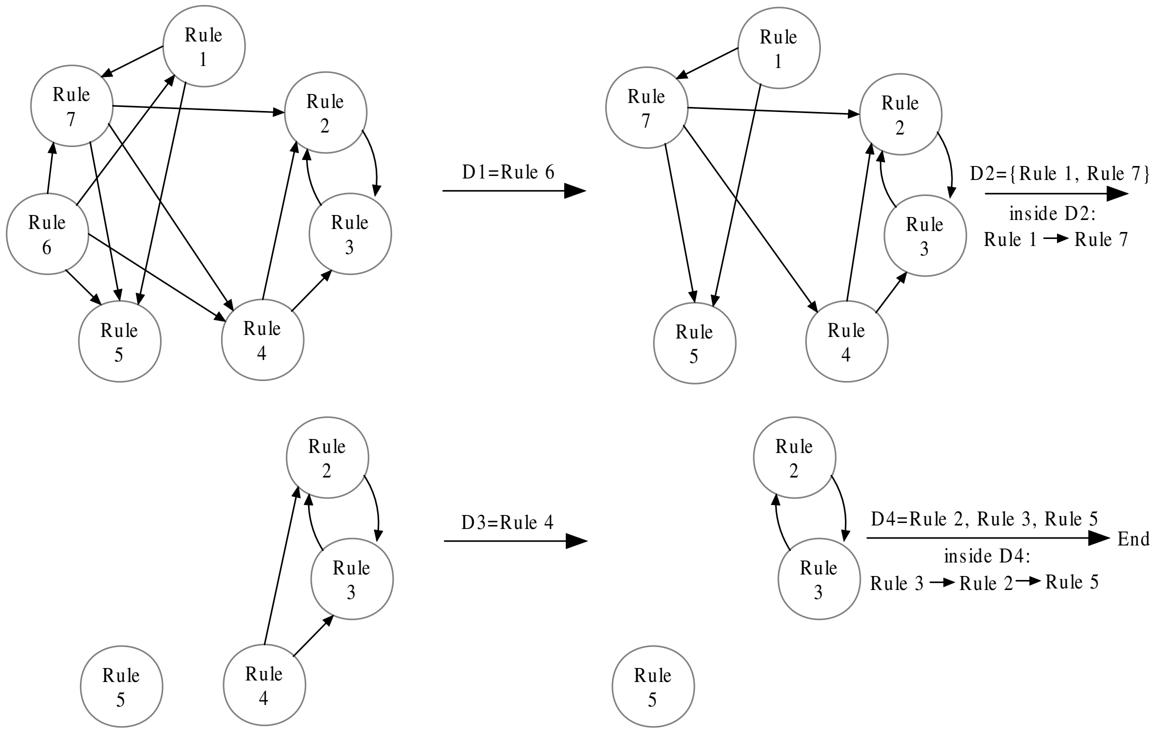

Fig. 1. Descending ranking steps

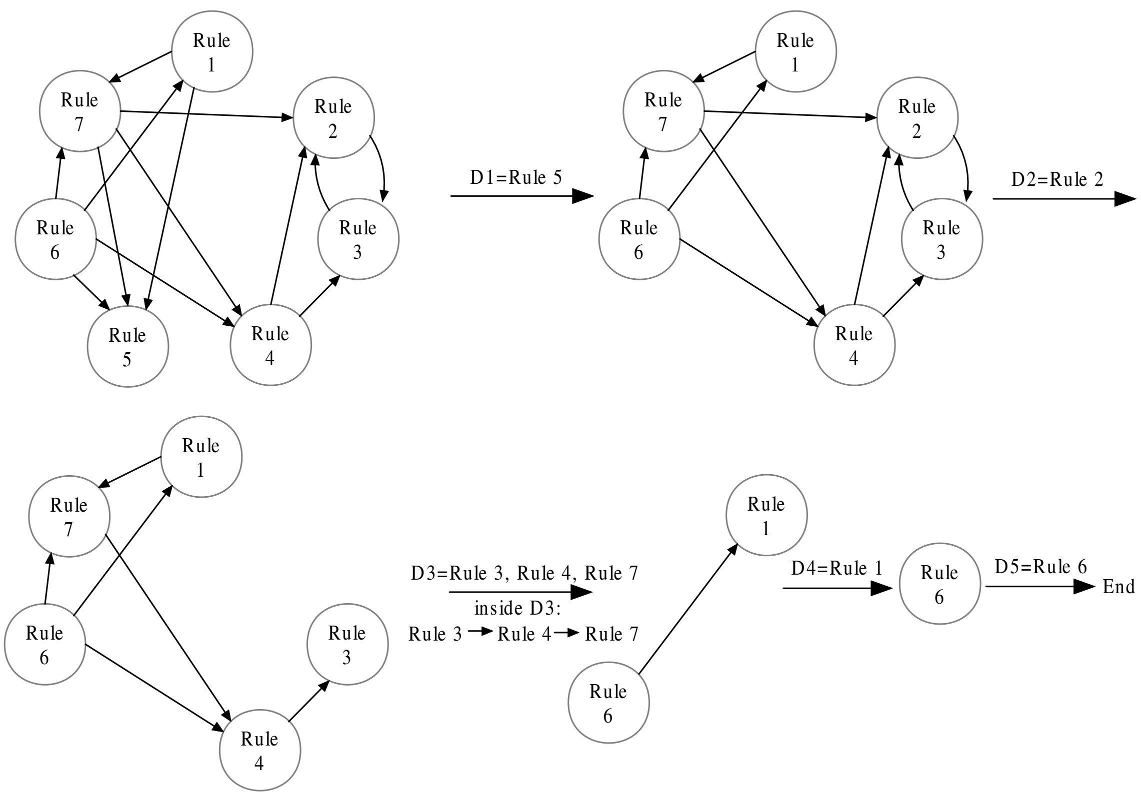

Fig. 2. Ascending ranking steps

Table 3. Ranking table

Alternatives	r'	r''	r	Ranking result of [2]
Rule 1	2	2	2	2
Rule 2	6	6	6	4
Rule 3	5	5	5	2
Rule 4	4	4	4	6
Rule 5	7	7	7	5
Rule 6	1	1	1	1
Rule 7	3	3	3	3

6 Discussion

ELECTRE-I, ELECTRE-II and ELECTRE-III methods have a lot in common for the outranking problem. Based on the definition of the integrative outranking relation, the new ELECTRE method has been proposed. It is the extension of ELECTRE-I, ELECTRE-II and ELECTRE-III methods. The outranking relations in those three methods can be considered as three special situations of this integrative outranking relation.

6.1 Comparison with ELECTRE-I

When the concordance index is built in a linear form and both the thresholds $q_j(g_j)$ and $p_j(g_j)$ equal to 0, a crisp index that is the same as in ELECTRE-I has been defined:

$$S_j(a,b) = \begin{cases} 1, & g_j(a) \geqslant g_j(b) \\ 0, & g_j(a) < g_j(b) \end{cases} .$$

The discordance index as in ELECTRE-I comes out when it has been built in a linear form and the discordance threshold $u_j(g_j)$ equals to the veto threshold $v_j(g_j)$, i.e.,

$$d_j(a,b) = \begin{cases} 0, & g_j(b) - g_j(a) < v_j(g_j(a)) \\ 1, & g_j(b) - g_j(a) \geqslant v_j(g_j(a)) \end{cases} .$$

The overall concordance and discordance relations are defined in the same way between the ELECTRE-I and the integration method. The procedure of combining these two indices to form the outranking relation is the same if the preponderance index $C'(a,b)$ is not taken into account.

6.2 Comparison with ELECTRE-II

In ELECTRE-II method, there are multiple levels of the concordance condition and discordance condition that are specified to construct two outranking relations (strong and weak outranking relation). Based on the discussion about the concordance relation $C(a,b)$ and the introduction of three thresholds ($0 < p^- < p^0 < p^*$) to indicate three levels, the same definition of overall concordance relation as in ELECTRE-II has been obtained from the new overall concordance relation. In ELECTRE-II method, there are also three levels of discordance index: the high level, the middle level and the low level. They are defined by two parameters $q_j^0(g_j)$ and $q_j^*(g_j)$. Corresponding to that, three levels of the overall non-discordance relation $ND(a,b)$ are formed. $ND(a,b) = 1$ and $ND(a,b) = 0$ imply a low and a high level on discordance respectively. If $ND(a,b)$ takes the value in $(0,1)$, it implies a middle level. The discordance index and the overall non-discordance relation can be respectively defined as:

$$d_j(a,b) = \begin{cases} 0, & g_j(b) - g_j(a) \leqslant q_j^0(g_j(a)) \\ \dfrac{g_j(b) - (g_j(a) + q_j^0(g_j(a)))}{q_j^*(g_j(a)) - q_j^0(g_j(a))}, & q_j^0(g_j(a)) \leqslant g_j(b) - g_j(a) \leqslant q_j^*(g_j(a)) \\ 1, & g_j(a) - g_j(b) \geqslant q_j^*(g_j(a)) \end{cases} ,$$

$$ND(a,b) = \prod_{j=1}^{n} (1 - d_j(a,b)).$$

In the new procedure, by treating the discordance threshold $u_j(g_j)$ and the veto threshold $v_j(g_j)$ as the two parameters $q_j^0(g_j)$ and $q_j^*(g_j)$ to indicate the high level and the low level respectively, the same discordance index as in ELECTRE-II is obtained. Based on these definitions, the strong outranking relation and the weak outranking relation are obtained. The strong outranking relation aFb is defined if and only if these conditions are hold:

$$C(a,b) > p^*,\ C'(a,b) = 1 \text{ and } ND(a,b) \in (0,1],$$

or

$$C(a,b) > p^0,\ C'(a,b) = 1 \text{ and } ND(a,b) = 1.$$

The weak outranking relation afb is defined when this condition is hold:

$$C(a,b) > p^-,\ C'(a,b) = 1 \text{ and } ND(a,b) = (0,1].$$

Hence, the strong and weak relations can be obtained by controlling the value of the integrative fuzzy relation $S(a,b)$.

6.3 Comparison with ELECTRE-III

The valued outranking relation as in ELECTRE-III is obtained when the conditions are hold: (1) the concordance index and discordance index are defined in linear way; (2) the discordance threshold $u_j(g_j)$ should be defined so that the difference $\varDelta$ is as small as possible on average:

$$\varDelta = \left| \prod_{j\in\overline{F}} \frac{1-d_j(a,b)}{1-C(a,b)} - \prod_{j\in F}(1-d'_j(a,b)) \right|,$$

where, in order to distinguish, we let $d_j(a,b)$ denote the discordance index in ELECTRE-III method and $d'_j(a,b)$ denote the discordance index in the integrative ELECTRE; (3) The preponderance index $C'(a,b)$ is ignored.

7 Conclusions

In this paper, a comprehensive and integrative outranking relation has been proposed. It is the extension of the outranking relations in those three methods, including most information of them. But the information included in the integrative relation is more abundant and the parameters in this method is more flexible to be controlled. Particularly, the correlation between the concordance and the discordance has been eliminated by bringing in a new parameter $u_j(g_j)$. Based on the integrative outranking relation, those three ELECTRE methods have been integrated. This method would be useful because it avoids the difficulty of choosing among those three ELECTRE methods when the DM deals with ranking problem. Moreover, this integration makes this new ELECTRE more suitable for the uncertain problem. The illustrative example given in this paper has proved the rationality of this method for ranking problem.

Because the data considered in the decision problem might be imprecise or uncertain, we have defined the concordance index and discordance index in fuzzy way in this paper. Then, a fuzzy outranking relation has been obtained. But actually, the data in this paper are denoted in a certain way. As a further research area, we can consider the data to be interval numbers or fuzzy numbers or the other uncertain forms. In addition, DM may have a vague understanding of what the parameters represent and the point of view can evolve during the integrating process. Moreover, in group decisions, lack of consensus among DMs can be also a critical issue. Hence, further work should be pursued in order to propose a more comprehensive and useful ELECTRE method to support the DM's decision in the more imprecise or uncertain situation.

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