ISSN 1746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 1, No. 2, 2005, *pp*. 99-104

Simulation research on air-burst dispersion of some multiple launch rocket system

Xuefeng Zhang [+], Cungui Yu, Dawei Ma

School of Mechanical Engineering, NJUST, Nanjing, 210094, P.R.China

(*Received August 26 2005, accepted October 5 2005*)

Abstract. In order to research the air burst dispersion (ABD) of a new multiple launch rocket system, a simulation method is presented in this paper, which is concentrate on analyzing the influence of various random factors on ABD. First a launch dynamic model is established to simulate the initial disturbance that affected by impact force of gas. Then the influence of combustion-gas flow upon back projectile is analyzed with the impact force model of combustion-gas flow in ballistic coordinates system. Finally, the simulation result of ABD is obtained by solving the extern trajectory equations with Monte-Carlo method. The simulation result is validated by experimental data and shows that the simulation models are accurate and credible.

Keywords: multiple launch rocket system, dispersion, combustion-gas flow, simulation

1. Introduction

Dispersion is a very important standard to evaluate the capability of multiple launch rocket system (MLRS). According to the ballistic characteristics, dispersion can be sorted into vertical-target dispersion, air-burst dispersion (ABD) and ground dispersion [1]. With improvement of whole design precision, MLRS is applied into the modern anti-aircraft campaign system gradually. To evaluate the damage effectiveness of MLRS intercepting aircraft, ABD is a very important factor to be considered, because it can directly affect the hit probability of one salvo with six to eight rockets. Now, ABD of MLRS can be measured by three-dimensional radar when in dispersion test. However, this test method needs much ammunition consumption and is easily affected by weather, and especially, it can not reflect the influence of various factors upon ABD in detail. So, computer simulation technique would be much helpful.

For such a complicated model of MLRS, there are many random and nonrandom factors that affect ABD, such as flurry, initiative disturbance, thrust eccentricity, mass eccentricity and projectile weight deviation [2], etc. More over, in order to enhance the hit probability, a new MLRS increase the firing rate to the utmost extent. Thus, because of very short interval (about 0.2s) between two salvos, influence of combustion-gas flow of front projectile to trajectory of back projectile could not be ignored. That makes it difficult to simulate the influence of those factors upon ABD. To solve this difficulty, Monte Carlo analysis method can do a great job. Monte Carlo analysis method is a kind of numerical calculation method to find the approximate solution of mathematic, physics and engineering questions through random model and statistical test. In the process of simulating ABD of MLRS by Monte Carlo analysis method, we first produce the table of random numbers by random number generator according to the average and variance of those factors. Then, we can gain the simulation result of ABD by solving exterior trajectory equation with those random numbers.

2. Analysis on air-burst dispersion

ABD is a measurement on dispersion of MLRS when intercepting aircraft. It denotes the deviation of one collision point from average collision point. The main factors that influence ABD are:

(1) Deviation of initial velocity;
(2) Initial disturbance;

[+] Corresponding author.
Email address: zxf_5218@163.com

(3) Gust wind;
(4) Mass eccentricity and dynamic unbalance;
(5) Thrust misalignment;

The probability deviations of those above factors are needed to simulate the ABD of this system with Monte Carlo method. For initial velocity, thrust misalignment, mass eccentricity and dynamic unbalance, we can obtain the probability deviations by test. For gust wind, we can use the common data [3]. However, initial disturbance would be greatly influenced by combustion-gas flow because of very short interval. Initial disturbance contributes much to the ABD of MLRS, so accurate initial disturbance simulation is necessary to evaluate the firing accuracy scientifically in system modeling.

3. Influence of combustion-gas flow upon initial disturbance

It is a pivotal step that we establish the accurate launch dynamic model when studying the initial disturbance of MLRS. Launch dynamics has been greatly developed in recent years. That would enable us to establish more accurate equation for initial disturbance, and would be a favorable condition for the simulation of dispersion of MLRS.

In this paper, a three-dimensional solid model is established with ADAMS simulation software according to the actual size and structure of the rocket system. This model provides the quantity of components, position of mass center and moment of inertia for dynamics analysis.

Suppose the viscosity of combustion-gas flow can be ignored, and the gas is fit for ideal gas equation of state. In body coordinates system of (ξ,η,ζ), the time related intense conservation form of 3D compressible flow Euler equation [4] is

$$\hat{Q}_t+\hat{F}_\xi+\hat{G}_\eta+\hat{H}_\zeta=0 \tag{1}$$

In (1) $\hat{Q}=Q/J$, $\hat{F}=(\xi_x F+\xi_y G+\xi_z H)/J$, $\hat{G}=(\eta_x F+\eta_y G+\eta_z H)/J$, $\hat{H}=(\zeta_x F+\zeta_y G+\zeta_z H)/J$, $Q=[\rho,\rho u,\rho v,\rho w,\rho e]^T$, $F=[\rho u,\rho u^2+p,\rho uv,\rho uw,u(e+p)]^T$, $G=[\rho u,\rho uv,\rho v^2+p,\rho uw,v(e+p)]^T$, $H=[\rho u,\rho uw,\rho uw,\rho w^2+p,w(e+p)]^T$. $\hat{F}$ 、 $\hat{G}$ and $\hat{H}$ are vectors of gas flux. $J=|\partial(\xi,\eta,\zeta)/\partial(x,y,z)|$.

According to the finite bulk discrete form of equation (1)

$$\hat{Q}^{n+1}_{i,j,k}=\hat{Q}^{n}_{i,j,k}+\frac{\Delta t}{\Delta\xi}\left[\hat{F}^{n}_{i+1/2,j,k}-\hat{F}^{n}_{i+1-2,j,k}\right]+\frac{\Delta t}{\Delta\eta}\left[\hat{G}^{n}_{i,j+1/2,k}-\hat{G}^{n}_{i,j-1/2,k}\right]+\frac{\Delta t}{\Delta\xi}\left[\hat{H}^{n}_{i,j,k+1/2}-\hat{H}^{n}_{i,j,k-1/2}\right] \tag{2}$$

we compute the impacting flow field of rocket. As shown in Fig. 1.

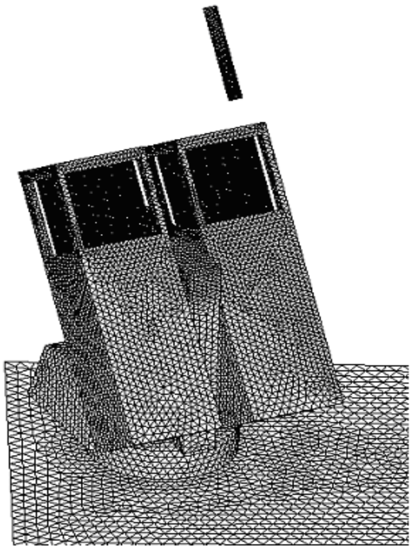


Fig.1. Modeling of launch dynamics on MLRS

When establishing the dynamic model, we treat bodywork, revolution body and elevating part as rigid body subsystem that restrain elastically with each other. The structure's flexibility effect is simulated by spring and damper which connect all the rigid bodies. The elastic coefficients and damping coefficients were gained by experiment. According to the foregoing computed result of combustion-gas flow, we add impact force of gas, locking force and lateral force of guide lug to the model. And then, the vibration curves of directional nozzle are worked out. As shown in Fig. 2, Fig. 3 and Fig. 4. From these figures we can find that the vibration of nozzle would only attenuate to one third of the max vibration velocity in time 0.2s. It would bring quite influence upon ABD. With these simulation data, the average and variance of initial disturbance are obtained for the following simulation.

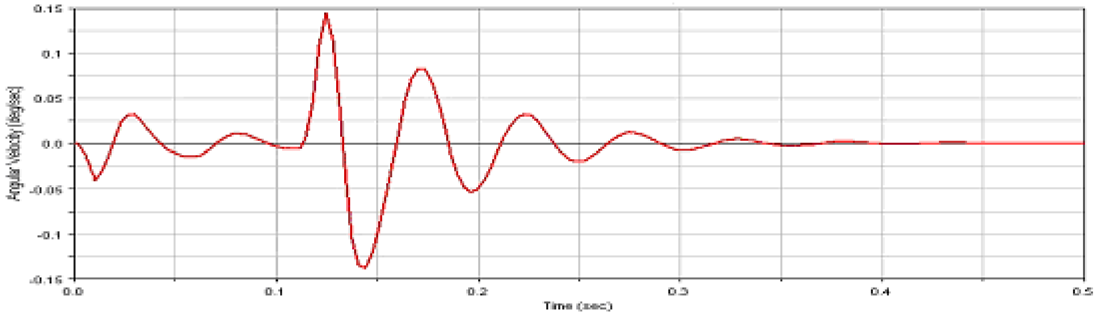


Fig.2. Angular velocity of nozzle in x direction

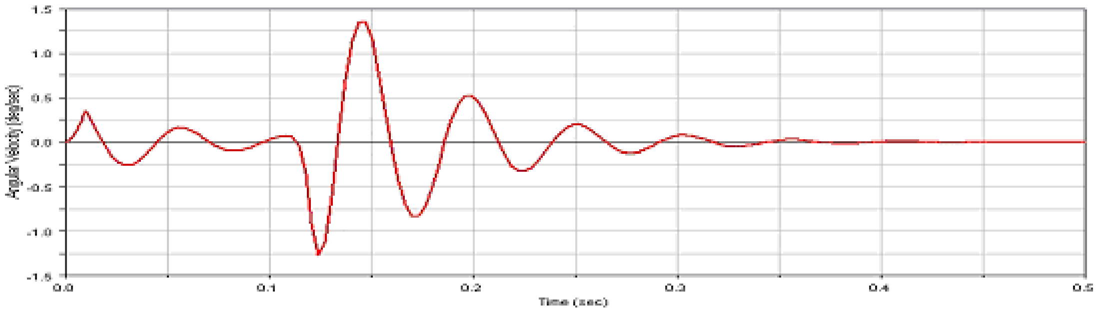


Fig.3. Angular velocity of nozzle in y direction

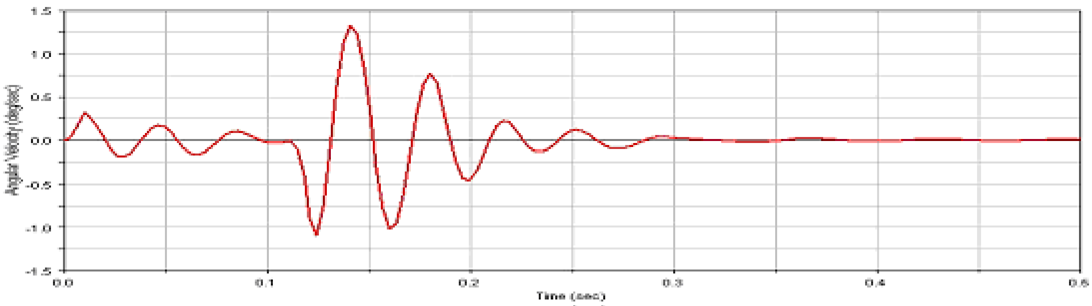


Fig.4. Angular velocity of nozzle in z direction

4. Influence of combustion-gas flow upon trajectory

Commonly, the shooting interval of a MLRS is 0.5s. The front projectile is more than 50m away from the back projectile when the back projectile is shot. The force of combustion-gas flow of front projectile to back projectile would be nearly zero and could be ignored [1]. In order to enhance the hit probability, the shooting interval of a new anti-aircraft MLRS was decreased to 0.2s. Thus, the front projectile and the back projectile would be 8m apart in upright distance and 1.5m apart in transverse distance when the back projectile is shot. The force of combustion-gas flow of front projectile to back projectile could not be ignored.

Set F_B is the resultant force of the combustion-gas flow of front projectile to back projectile and F_B acts at D. As shown in Fig. 5(a). At center of mass (CM), we can add two forces, F'_B and F''_B, which are equal in size but contrary in direction. They are equal to F_B and their directions are parallel with F_B. That won't change the flight state of rocket. Thus, the rocket can be regard as being impacted by F'_B and the moment of couple which is composed of F''_B and F_B. We define M_B as the moment of couple. As shown in Fig. 5(b). Then we resolve F'_B in ballistic coordinates system and obtain three coordinate components: F_{Bx} (parallel with velocity), F_{By} (parallel with lifting force), F_{Bz} (parallel with direction of horizontal). The coordinate transformation matrix from ballistic coordinates system to body coordinates system [3] is

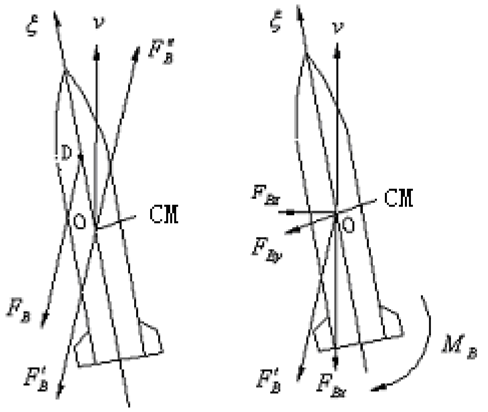
$$A=\begin{bmatrix}\cos\delta_1\cos\delta_2 & \cos\delta_2\sin\delta_1 & \sin\delta_2\\ -\sin\delta_1 & \cos\delta_1 & 0\\ -\sin\delta_2\cos\delta_1 & -\sin\delta_2\sin\delta_1 & \cos\delta_2\end{bmatrix}$$

Then coordinate components of F_B in body coordinates system are

$$\begin{bmatrix}F_{B\xi}\\ F_{B\eta}\\ F_{B\zeta}\end{bmatrix}=A\begin{bmatrix}F_{Bx}\\ 0\\ 0\end{bmatrix}+A\begin{bmatrix}0\\ F_{By}\\ 0\end{bmatrix}+A\begin{bmatrix}0\\ 0\\ F_{Bz}\end{bmatrix}=\begin{bmatrix}\cos\delta_1\cos\delta_2F_{Bx}+\cos\delta_2\sin\delta_1F_{By}+\sin\delta_2F_{Bz}\\ -\sin\delta_1F_{Bx}+\cos\delta_1F_{By}\\ -\sin\delta_2\cos\delta_1F_{Bx}-\sin\delta_2\sin\delta_1F_{By}+\cos\delta_2F_{Bz}\end{bmatrix}$$

Set L is the length of $\overline{OD}$. Then coordinate components of M_B in body coordinates system are

$$\begin{bmatrix}M_{B\xi} & M_{B\eta} & M_{B\zeta}\end{bmatrix}^T=\begin{bmatrix}0 & F_{B\zeta}L & F_{B\eta}L\end{bmatrix}^T \tag{3}$$



(a) (b)

Fig.5. Analysis on impact force of combustion-gas flow upon back projectile

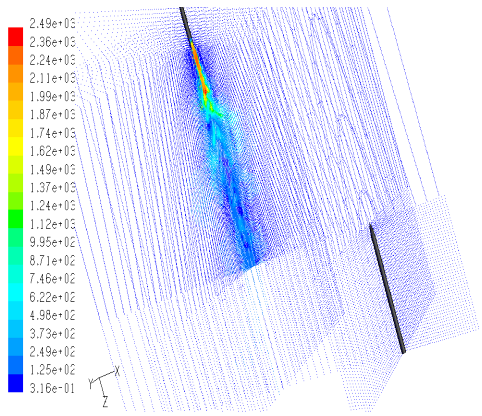


Fig.6. Simulation model of impact force of combustion-gas flow upon back projectile

Function (3) shows that $M_{B\eta}$ and $M_{B\zeta}$ can be expressed by F_{Bx}, F_{By}, F_{Bz} and L. Thus we can obtain F_{Bx}, F_{By}, F_{Bz}, $M_{B\eta}$, $M_{B\zeta}$ as long as the coordinate components and acting position of F_B in ballistic coordinates system are obtained. Upon that, we establish the impact force model of combustion-gas flow upon back projectile in ballistic coordinates system, as shown in Fig. 6, and compute with impact flow field of combustion-gas. The results are shown in table 1.

Table 1. Simulation results of F_x, F_y, F_z and L

t/s	0.0	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.60
F_x/n	10.05	7.71	6.23	5.04	4.22	3.65	2.41	1.82	1.27	0.92	0.45	0.29	0.10	0.03	0.0	0.0
F_y/n	2.17	1.71	1.39	1.12	0.83	0.70	0.51	0.33	0.18	0.09	0.01	0.0	0.0	0.0	0.0	0.0
F_z/n	4.16	3.40	2.81	2.19	1.63	1.36	1.02	0.78	0.61	0.40	0.28	0.21	0.12	0.03	0.01	0.0
L/m	0.11	0.13	0.12	0.14	0.15	0.16	0.17	0.165	0.175	0.20	0.213	0.22	0.22	0.23	0.23	0.24

According to above-mentioned analysis of impact force of combustion-gas flow, we establish external trajectory equations of back projectile, which is based on the six-degree-of-freedom rigid body trajectory equations in [3].

$$\begin{cases} m\dfrac{dv}{dt} = F_p \cos\delta_1 \cos\delta_2 - \dfrac{1}{2}\rho S C_x v_r (v - W_{\parallel}) + \dfrac{1}{2}\rho S C_y' [W_{x\perp}{}^2 + W_z^2 - W_{x\perp}(v - W_{\parallel})\delta_1 + W_z \delta_2] - mg\sin\theta_1 + F_{Bx} \\ mv\cos\psi_2 \dfrac{d\theta_1}{dt} = F_p \cos\delta_2 \sin\delta_1 - \dfrac{1}{2}\rho S C_x v_r W_{x\perp} + \dfrac{1}{2}\rho S C_y' [v_r{}^2 \delta_1 - (v - W_{\parallel}) W_{x\perp}] - mg\cos\theta_1 + F_{By} \\ mv\dfrac{d\psi_2}{dt} = F_p \sin\delta_2 + \dfrac{1}{2}\rho S C_x v_r W_x + \dfrac{1}{2}\rho S C_y' [v_r{}^2 \delta_2 + (v - W_{\parallel}) W_z] + mg\sin\theta_1 \sin\psi_2 + F_{Bz} \\ C\dfrac{d\omega_\xi}{dt} = M_{xp} + \dfrac{1}{2}\rho v_r{}^2 S l m_{xw} + M_{B\xi} \\ A\dfrac{d\omega_\eta}{dt} = -\dfrac{1}{2}\rho S l m_z' v_r [(v - W_{\parallel})\delta_2 + W_z] + \dfrac{1}{2}\rho S l m_y' v_r [(v - W_{\parallel})\delta_1 - W_{x\perp}] - C\omega_\xi \omega_\zeta + A\omega_\zeta{}^2 tg\varphi_2 + M_{B\eta} \\ A\dfrac{d\omega_\zeta}{dt} = \dfrac{1}{2}\rho S l m_z' v_r [(v - W_{\parallel})\delta_1 - W_{x\perp}] + \dfrac{1}{2}\rho S l m_y' v_r [(v - W_{\parallel})\delta_2 - W_z] + C\omega_\xi \omega_\eta - A\omega_\eta \omega_\zeta tg\varphi_2 + M_{B\zeta} \\ \dfrac{d\varphi_a}{dt} = \dfrac{1}{\cos\varphi_2}\omega_\xi \\ \dfrac{d\varphi_2}{dt} = -\omega_\eta \\ \dfrac{d\gamma}{dt} = \omega_\xi - \omega_\zeta tg\varphi_2 \\ \dfrac{dx}{dt} = v\cos\psi_2 \cos\theta_1 \\ \dfrac{dy}{dt} = v\cos\psi_2 \sin\theta_1 \\ \dfrac{dz}{dt} = v\sin\psi_2 \\ \delta_1 = \varphi_a - \theta_1 \\ \delta_2 = \varphi_2 - \psi_2 \end{cases}$$

5. Simulation research on air-burst dispersion

According to the above simulation results of impact force model of combustion-gas flow and extern trajectory equations, we establish the simulation system based on MATLAB and work it out with Monte-Carlo method. The concrete steps are [5]:

Generating pseudo-random sequence that fit for the distributing characteristic of those random factors.

Computing the launch dynamic model by using the above-mentioned pseudo-random sequence and structure parameters, to obtain initial disturbance and then provide initial parameters for computation of extern trajectory.

Computing extern trajectory equations with aerodynamic parameters and initial parameters obtained in (2) to work out the coordinate of a burst point (x, y, z).

Repeating (1)、(2)、(3) for n times (n satisfies the requirement for precision of Monte Carlo method), and thus gain n groups of simulated data of burst points (x_1, y_1, z_1), (x_2, y_2, z_2), $\cdots$, (x_n, y_n, z_n).

Estimating the ABD of MLRS with n groups of simulated data of burst points (x_1, y_1, z_1), (x_2, y_2, z_2), $\cdots$, (x_n, y_n, z_n), and we obtain

$$\hat{E}_x = 0.6745 \times \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$\hat{E}_y = 0.6745 \times \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}}$$

$$\hat{E}_z = 0.6745 \times \sqrt{\frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1}}$$

Here, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$.

The simulation result is shown in table 2 and figure 7. By comparison with experimental data which is logged in firing range test, we can find that the simulation result is credible and the simulation model is accurate.

6. Conclusions

This paper presents a simulation method to study the influence of random factors, such as combustion-gas flow and initial disturbance, upon extern trajectory by establishing simulation models. According to the model, we simulate the ABD of this MLRS by using Monte Carlo analysis method. The simulation results are verified by experimental data and show that the simulation models are accurate and rational. Further more, these simulation models and results would be a theoretical support for further study on damage effect of this MLRS.

Table 2. Comparison of simulation data with experimental data

	Simulation data	Experimental data	Relative error (%)
$E_{\bar{x}}$ /m	41.97	39.86	5.29
$E_{\bar{y}}$ /m	48.49	45.31	7.02
$E_{\bar{z}}$ /m	109.62	118.62	-7.59

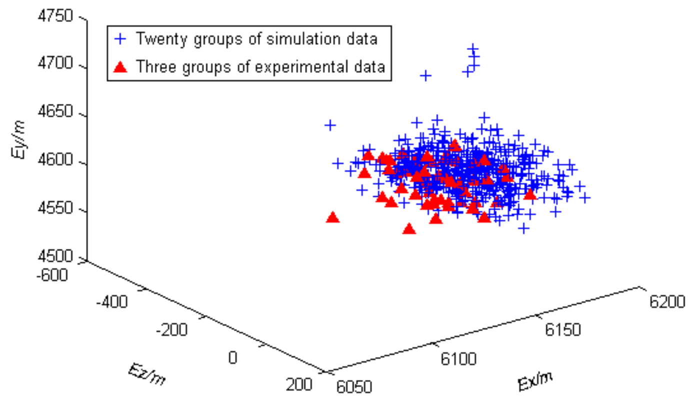


Fig.7 Simulation result and test result of burst points

7. Acknowledgements

We would like to thank all the teachers in Institute of Rocket Launcher of NJUST. They provide all kinds of parameters of the rocket launcher and give many helpful suggestions to the simulation models.

8. References

[1] Xifu Guo, *Firing accuracy analysis for long range gun weapon systems*, National Defence Industry Press, 2004.

[2] Guoping Wang, Xiaoting Rui, Weidong Chen, *Dispersion simulation technique of multiple tube weapon*, Journal of System Simulation, 16(2004), pp. 963-966.

[3] Mingyou Xu, *Rocket exterior ballistics*, Weaponry Industry Press, 1989.

[4] S. Osher, S. Chakravarthy, *Upwind schemes and boundary conditions with applications to Euler equations in general coordinates*, Journal of Computational Physics, 50(1983), pp. 447-481.

[5] Changsheng Zhou, Xiuchao Zhao, Guangshen Qiu, *The test analysis of affection of spin rocket projectile's unbalance to the dispersion*, Journal of Ballistics, 10(1998), pp. 59-67.