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Fuzzy adaptive H_∞ control for a class of uncertain nonlinear time-delay systems *

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Abstract. Combining both kinds of fuzzy logic forms including fuzzy T-S model and adaptive fuzzy logic systems, this paper presents an observer-based H_∞ control scheme for a class of uncertain nonlinear time-delay systems. Firstly, the fuzzy T-S model is used to approximate the nonlinear systems, and an observer is designed to observe the system states, and the fuzzy control law of the fuzzy model is derived by the LMI. Secondly, the adaptive time-delay fuzzy logic systems are constructed, and the modeling errors and the uncertain nonlinear parts are eliminated by a compensator based on the adaptive time-delay fuzzy logic systems with three adjustable parameters: weights, centers and widths. It is proved that the closed loop system satisfies the anticipant H_∞ performance, The simulation results demonstrate that the control scheme is effective.

Keywords: fuzzy T-S model, adaptive fuzzy logic systems, nonlinear systems, time-delay.

1. Introduction

In engineering, the existence of time-delay and uncertainties deteriorates the system performance. Time-delay and uncertainties pose great difficulties to the stabilization control design. Therefore, the problem of controller design for uncertain nonlinear time-delay systems has attracted considerable attention.

A typical approach of effective control for nonlinear time-delay systems is the local fuzzy-T-S- model-based linearization approach, and this approach has been successful applied in [1-2]. However, the modeling error is neglected in these studies. Therefore, the designed controller does not always guarantee the stability of the original system. Besides, in nonlinear modeling, the paper [3] considers that the modeling error has the upper bound, the papers [4-5] consider that the modeling error satisfies the matching condition, However, the upper bound and the matching condition are difficult to find in practice. The paper [6] designs a neural network compensator to compensate the modeling error. With regard to the uncertain nonlinearities in nonlinear systems, it is often required to satisfy the certain constraint in [7]. In fact, it isn't easy to look for the constraint condition. Meanwhile, the system states cannot be often directly measured in engineering. It is necessary to design the observer-based H_∞ control scheme.

The adaptive fuzzy logic systems have the universal approximation property and could uniformly approximate nonlinear continuous functions to an arbitrary accuracy. The adaptive fuzzy logic systems could sufficiently make use of the linguistic information and the expert information. The adaptive fuzzy logic systems are used to model uncertain nonlinear systems by a set of fuzzy "if-then" rules. When a proper control is given to the model, an anticipant output is produced from the nonlinear systems. At present, the adaptive fuzzy logic systems have been successfully used in nonlinear control [8-9].

Combining fuzzy T-S model and adaptive fuzzy logic systems, this paper presents a new H_∞ control scheme for a class of uncertain nonlinear time-delay systems. Firstly, the fuzzy T-S model is used to approximate the nonlinear systems, and the fuzzy controller is designed by the linear matrix inequalities to guarantee the stability of the fuzzy system. Secondly, the adaptive time-delay fuzzy logic systems are

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constructed, and the modeling error and the uncertain nonlinearities are eliminated by a compensator based on the adaptive time-delay fuzzy logic systems with three adjustable parameters: weights, centers and widths. It is proved that the closed-loop system satisfies the anticipant H_∞ performance. The simulation results demonstrate that the control scheme is effective.

2. Problem formulation

Consider the following uncertain nonlinear time-delay system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(x, x(t-\tau_1), \cdots, x(t-\tau_r), u) + \tilde{f}_2(x, x(t-\tau_1), \cdots, x(t-\tau_r), u) + d_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_4(x, x(t-\tau_1), \cdots, x(t-\tau_r), u) + \tilde{f}_4(x, x(t-\tau_1), \cdots, x(t-\tau_r), u) + d_4 \end{cases} \quad (1)$$
$$y = Cx$$

where $x_1, x_2 \in R^{n_1}, x_3, x_4 \in R^{n_2}$ are the measurable state vectors, $u \in R^m$ is the control input vector, f_2, f_4 are the known smooth nonlinear functions, $\tilde{f}_2, \tilde{f}_4$ are the unknown uncertain nonlinearities of the system. τ_i (for $i = 1,2,\cdots,r$) are the time delays. d_2, d_4 denote the external disturbance. Denote $x = [x_1^T, x_2^T, x_3^T, x_4^T]^T \in R^n$, $n = 2m, m = n_1 + n_2$, $d = [0, d_2^T, 0, d_4^T]^T$.

The known part of the system (1) can be approximated by a fuzzy T-S model composed of L rules. The ith rule of fuzzy model is in the following form: If $z_1(t)$ is F_1^i and, $\cdots$, and $z_s(t)$ is F_s^i Then

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \sum_{l=1}^{r} A_{il} x(t-\tau_l) + B_i u(t) + d \\ y(t) &= Cx(t) \end{aligned} \quad , \; i = 1,2,\cdots,L \quad (2)$$

where $z_1(t), \cdots, z_s(t)$ are the premise variables, F_j^i (for $j = 1,2,\cdots,s$) are the fuzzy sets, L is the number of If-Then rules, A_i , B_i and A_{il} are some constant matrices with compatible dimensions.

$B_i = \begin{bmatrix} 0 & b_{i1}^T & 0 & b_{i2}^T \end{bmatrix}^T \in R^{n \times m}$, $b_{i1} \in R^{n_1 \times m}, b_{i2} \in R^{n_2 \times m}$. The final output of the fuzzy system is inferred as follows

$$\dot{x}(t) = \sum_{i=1}^{L} \mu_i [A_i x(t) + \sum_{l=1}^{r} A_{il} x(t-\tau_l)] + \sum_{i=1}^{L} \mu_i B_i u(t) + d \quad (3.1)$$

$$y(t) = Cx(t) \quad (3.2)$$

where $\mu_i = v_i(z(t)) \Big/ \sum_{i=1}^{L} v_i(z(t))$, $v_i(z(t)) = \prod_{j=1}^{s} F_j^i(z_j(t))$, $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i . It is easy to find that $\mu_i \geq 0$, for $i = 1,2,\cdots,L$ and $\sum_{i=1}^{L} \mu_i = 1$ for all t . Therefore, the modeling error and the uncertain nonlinearities of the nonlinear system (1) can be expressed as

$$B\Delta(x, x(t-\tau_1), \cdots, x(t-\tau_r)) = \begin{bmatrix} 0 \\ \Delta f_2 \\ 0 \\ \Delta f_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ f_2 + \tilde{f}_2 \\ x_4 \\ f_4 + \tilde{f}_4 \end{bmatrix} - (\sum_{i=1}^{L} \mu_i [A_i x(t) + \sum_{l=1}^{r} A_{il} x(t-\tau_l)] + \sum_{i=1}^{L} \mu_i B_i u(t)) \quad (4)$$

where $B = \begin{bmatrix} 0 & I_{n_1} & 0 & 0 \\ 0 & 0 & 0 & I_{n_2} \end{bmatrix}^T$, $\Delta(x, x(t-\tau_1), \cdots, x(t-\tau_r)) = \begin{bmatrix} \Delta f_2 \\ \Delta f_4 \end{bmatrix}$.

For convenience, denote $\Delta(x, \tau) = \Delta(x, x(t-\tau_1), \cdots, x(t-\tau_r))$, therefore, the nonlinear system (1) could be rearranged as

$$\dot{x}(t) = \sum_{i=1}^{L} \mu_i [A_i x(t) + \sum_{l=1}^{r} A_{il} x(t-\tau_l)] + \sum_{i=1}^{L} \mu_i B_i u(t) + B\Delta(x, \tau) + d \quad (5.1)$$

$$y(t) = Cx(t) \tag{5.2}$$

3. The design of the output feedback controller

If we do not consider the effect of the $\Delta(x,\tau)$. Denote $\Delta(x,\tau)=0$ in (5.1). Design a controller to guarantee to stabilize the corresponding closed loop system of the linear part of the nonlinear system (5.1), and make the H_∞ performance satisfied. Because the state can not be measured, we design the observer-based output feedback controller.

The overall fuzzy observer and the overall feedback controller are given by

$$\dot{\hat{x}}(t) = \sum_{i=1}^{L}\mu_i[A_i\hat{x}(t) + \sum_{l=1}^{r}A_{il}\hat{x}(t-\tau_l)] + \sum_{i=1}^{L}\mu_i B_i u(t) + \sum_{i=1}^{L}\mu_i L_i(y(t)-\hat{y}(t)) \tag{6.1}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{6.2}$$

$$u(t) = \sum_{i=1}^{L}\mu_i K_i\hat{x}(t) \tag{6.3}$$

where L_i and K_i are matrixes with proper dimensions.

Denote $e(t) = x(t) - \hat{x}(t)$. Using (3.1) and (6) , we derive

$$\dot{e}(t) = \sum_{i=1}^{L}\mu_i[(A_i - L_iC)e(t) + \sum_{l=1}^{r}A_{il}e(t-\tau_l)] + d \tag{7}$$

Substituting (3.2) and (6.2) into (6.1), we derive

$$\dot{\hat{x}}(t) = \sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[(A_i + B_iK_j)\hat{x}(t) + \sum_{l=1}^{r}A_{il}\hat{x}(t-\tau_l)] + \sum_{j=1}^{L}\mu_i L_i Ce(t) \tag{8}$$

Denote $\tilde{x}(t) = [\hat{x}^T(t), e^T(t)]^T$, the closed-loop system is yielded

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\overline{A}_{ij}\tilde{x}(t) + \sum_{l=1}^{r}\overline{A}_{il}\tilde{x}(t-\tau_l)] + d' \tag{9}$$

where $\overline{A}_{ij} = \begin{bmatrix} A_i + B_iK_j & L_iC \\ 0 & A_i - L_iC \end{bmatrix}$, $\overline{A}_{il} = \begin{bmatrix} A_{il} & 0 \\ 0 & A_{il} \end{bmatrix}$, $d' = \begin{bmatrix} 0 \\ d \end{bmatrix}$.

Let us consider the H_∞ performance: For a prescribed attenuation $\rho > 0$, the H_∞ performance is achieved as

$$\int_0^T \tilde{x}^T(t)Q\tilde{x}(t)dt \le \tilde{x}^T(0)P\tilde{x}(0) + \sum_{l=1}^{r}\int_{-\tau_l}^{0}\alpha_l\tilde{x}^T(v)\tilde{x}(v)dv + \rho^2\int_0^T(d'^Td')dt \tag{10}$$

where P , Q are some symmetric and positive definite matrices, α_l (for $l = 1,2,\cdots,r$) are some positive scalars.

Theorem 1. There exists output feedback controller (6.3) so that the closed-loop system (9) satisfies the H_∞ performance (10), if there exist a symmetric and positive definite matrix P , a symmetric and positive definite matrix Q, and some positive scalars α_l (for $l = 1,2,\cdots,r$) satisfying

$$\overline{A}_{ij}^TP + P\overline{A}_{ij} + \sum_{l=1}^{r}\alpha_l^{-1}P\overline{A}_{il}\overline{A}_{il}^TP + \sum_{l=1}^{r}\alpha_l I + \frac{1}{\rho^2}PP + Q < 0 \quad (i,j = 1,\cdots,L) \tag{11}$$

Proof. Choose the *Lyapunov* function $V = \tilde{x}^T(t)P\tilde{x}(t) + \sum_{l=1}^{r}\int_{t-\tau_l}^{t}\alpha_l\tilde{x}^T(v)\tilde{x}(v)dv$

$$\begin{aligned}\dot{V} &= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) + \sum_{l=1}^{r}\alpha_l\tilde{x}^T(t)\tilde{x}(t) - \sum_{l=1}^{r}\alpha_l\tilde{x}^T(t-\tau_l)\tilde{x}(t-\tau_l) \\ &= (\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\overline{A}_{ij}\tilde{x}(t) + \sum_{l=1}^{r}\overline{A}_{il}\tilde{x}(t-\tau_l)])^TP\tilde{x}(t) + \tilde{x}^T(t)P(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\overline{A}_{ij}\tilde{x}(t) + \sum_{l=1}^{r}\overline{A}_{il}\tilde{x}(t-\tau_l)])\end{aligned}$$

$$+d'^T P\tilde{x}(t)+\tilde{x}^T(t)Pd' +\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t)\tilde{x}(t)-\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t-\tau_l)\tilde{x}(t-\tau_l)$$

$$\le(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\tilde{x}^T(t)\overline{A}_{ij}^T P\tilde{x}(t)+\tilde{x}^T(t)P\overline{A}_{ij}\tilde{x}(t)+\sum_{l=1}^{r}\alpha_l^{-1}\tilde{x}^T(t)P\overline{A}_{il}\overline{A}_{il}^T P\tilde{x}(t)+\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t-\tau_l)\tilde{x}(t-\tau_l)]$$

$$-(\frac{1}{\rho}P\tilde{x}(t)-\rho d')^T(\frac{1}{\rho}P\tilde{x}(t)-\rho d') +\rho^2 d'^T d'+\frac{1}{\rho^2}\tilde{x}^T(t)PP\tilde{x}(t)$$

$$+\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t)\tilde{x}(t)-\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t-\tau_l)\tilde{x}(t-\tau_l)$$

$$\le\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\tilde{x}^T(t)(\overline{A}_{ij}^T P+P\overline{A}_{ij} +\sum_{l=1}^{r}\alpha_l^{-1}P\overline{A}_{il}\overline{A}_{il}^T P+\sum_{l=1}^{r}\alpha_l I+\frac{1}{\rho^2}PP)\tilde{x}(t) +\rho^2 d'^T d'$$

From (12), we derive $\dot{V}\le-\tilde{x}^T(t)Q\tilde{x}(t) +\rho^2 d'^T d'$.

Integrating the above equation from $t=0$ to T yields (10). The proof is completed.

By the Schur complements, the inequalities (11) in Theorem 1 can be transformed into the linear matrix inequalities. Denote $P=diag\{P_1,P_2\}$, $Q=diag\{Q_1,Q_2\}$,where P_1、P_2、Q_1 and Q_2 are some symmetric and positive definite matrices. We derive

$$\overline{A}_{ij}^T P+P\overline{A}_{ij}+\sum_{l=1}^{r}\alpha_l^{-1}P\overline{A}_{il}\overline{A}_{il}^T P+\sum_{l=1}^{r}\alpha_l I+\frac{1}{\rho^2}PP+Q=\begin{bmatrix} S_{11} & P_1 L_i C \\ (L_i C)^T P_1 & S_{22}\end{bmatrix}<0 \tag{12}$$

where $S_{11}=P_1(A_i+B_iK_j)+(A_i+B_iK_j)^T P_1+\sum_{l=1}^{r}\alpha_l^{-1}P_1A_{il}A_{il}^T P_1+\sum_{l=1}^{r}\alpha_l I+\frac{1}{\rho^2}P_1P_1+Q_1$,

$$S_{22}=P_2(A_i-L_iC)+(A_i-L_iC)^T P_2+\sum_{l=1}^{r}\alpha_l^{-1}P_2A_{il}A_{il}^T P_2+\sum_{l=1}^{r}\alpha_l I+\frac{1}{\rho^2}P_2P_2+Q_2 .$$

Denote $W=P_1^{-1}$, $Y_j=K_jW$,（12）is equivalent to the linear matrix inequalities

$$\begin{bmatrix} S & W & L_iC \\ W & -(\sum_{l=1}^{r}\alpha_l I+Q_1)^{-1} & 0 \\ (L_iC)^T & 0 & S_{22}\end{bmatrix}<0 \tag{13}$$

where $S=A_iW+WA_i^T+B_iY_j+(B_iY_j)^T+\sum_{l=1}^{r}\alpha_l^{-1}A_{il}A_{il}^T+(\rho^2)^{-1}I$ .Denote $Z_i=P_2L_i$, $S_{22}<0$ is equivalent to the linear matrix inequalities

$$\begin{bmatrix} P_2A_i+A_i^T P_2-Z_iC-(Z_iC)^T+\sum_{l=1}^{r}\alpha_l I+Q_2 & P_2 \\ P_2 & -(\sum_{l=1}^{r}\alpha_l^{-1}A_{il}A_{il}^T+\frac{1}{\rho^2}I)^{-1}\end{bmatrix}<0 \tag{14}$$

Solving (14), we obtain P_2 and L_i (for $i=1,2,\cdots L$). And then, we obtain P_1 and K_i (for $j=1,2,\cdots,L$)by (13).

4. The compensator based on the adaptive fuzzy logic systems

The above analysis ignores the fuzzy modeling error and the uncertain nonlinearities. It is difficult to make the system(1) satisfy the anticipant H_∞ performance by only using the control law in Theorem 1. In this situation, we construct the time-delay fuzzy logic systems to eliminate the fuzzy modeling error and the uncertain nonlinearities. Therefore, the nonlinear system is stabilized and the desired H_∞ performance is achieved.

Combining the fuzzy output feedback control with the time-delay-fuzzy-logic-system-based control, we design the control law

$$u(t) = u_l(t) - u_f(t) \tag{15}$$

where $u_l(t)$ denotes the fuzzy control law in (6.3), $u_f(t)$ is the compensator based on the adaptive fuzzy logic systems.

Choose the adaptive compensator

$$u_f(t) = \begin{cases} E^{-1}\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta) & (16) \\ E^T(I+EE^T)^{-1}\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta) & (17) \end{cases}$$

If E is nonsingular, (16) is chosen, or else, (17) is chosen. where $\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta)$ is constructed by the adaptive fuzzy logic systems, $E_i = \begin{bmatrix} b_{i1} \\ b_{i2} \end{bmatrix} \in R^{m\times m}$, $E = \sum_{i=1}^{L} \mu_i E_i$.

Substituting (15) into (5.1) yields

$$\dot{x}(t) = \sum_{i=1}^{L} \mu_i [A_i x(t) + \sum_{l=1}^{r} A_{il} x(t-\tau_l)] + \sum_{i=1}^{L}\sum_{j=1}^{L} \mu_i \mu_j B_i K_j \hat{x}(t) - B(\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta) - \Delta(x,\tau)) + d \tag{18}$$

Denote $\tilde{x}(t) = [\hat{x}^T(t), e^T(t)]^T$, $\bar{B} = [0 \quad -B^T]^T$, where $e(t) = x(t) - \hat{x}(t)$. We get a new closed-loop system from (6) and (18)

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{L}\sum_{j=1}^{L} \mu_i \mu_j [\bar{A}_{ij}\tilde{x}(t) + \sum_{l=1}^{r} \bar{A}_{il}\tilde{x}(t-\tau_l)] + d' + \bar{B}(\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta) - \Delta(x,\tau)) \tag{19}$$

If the adaptive fuzzy logic systems $\hat{u}(\hat{x},\tau \mid \Theta,\alpha,\delta)$ could eliminate $\Delta(x,\tau)$, then the closed-loop system (19) is stable.

It has been proven that the adaptive fuzzy logic systems have the universal approximation property and could uniformly approximate nonlinear continuous functions to an arbitrary accuracy. We construct the adaptive time-delay fuzzy logic systems to approximate the time-delay vector function $\Delta(x,\tau)$ with m dimensions. The fuzzy logic systems have the form $\hat{\Delta}(x,\tau \mid \Theta,\alpha,\delta) = \Psi(x,\tau,\alpha,\delta)\Theta$, where $\Psi(x,\tau,\alpha,\delta) = diag[\xi_1(x,\tau,\alpha_1,\delta_1),\cdots, \ \xi_m(x,\tau,\alpha_m,\delta_m)]$, $\Theta = [\theta_1^T,\theta_2^T,\cdots,\theta_m^T]^T$, $\alpha = diag[\alpha_1,\cdots,\alpha_m]$, $\delta = diag[\delta_1,\cdots\delta_m]$, θ_i (for $i = 1,2,\cdots,m$) are the column vectors , α_i, δ_j (for $i, j = 1,2,\cdots,m$) are the row vectors. The weights Θ, the centers α and the widths δ are the adjustable parameters. The kth element in the vector function $\Delta(x,\tau)$ is in the form

$$y(x,\tau) = \sum_{j=1}^{M} \bar{y}^j (\prod_{i=1}^{n} \mu_{F_i^j}(x_i,\tau,\alpha_{kj},\delta_{kj})) \Big/ \sum_{j=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i,\tau,\alpha_{kj},\delta_{kj}))$$

where $\mu_{F_i^j}(x_i,\tau,\alpha_{kj},\delta_{kj}) = \mu_{F_i^j}(x_i,\alpha_{kj},\delta_{kj})\mu_{F_i^j}(x_i(t-\tau_1),\alpha_{kj},\delta_{kj}))\cdots\mu_{F_i^j}(x_i(t-\tau_r),\alpha_{kj},\delta_{kj})$ $\mu_{F_i^j}(x_i,\alpha_{kj},\delta_{kj})$ is the membership function. We have the following Theorem.

Theorem 2. Define the estimation errors of the parameter vector and the parameter matrix $\tilde{\Theta} = \Theta - \Theta^* = (\theta_i - \theta_i^*)_{m\times 1}$, $\tilde{\alpha} = \alpha - \alpha^* = diag(\alpha_1 - \alpha_1^*,\cdots,\alpha_m - \alpha_m^*)$ $\tilde{\delta} = \delta - \delta^* = diag(\delta_1 - \delta_1^*,\cdots,\delta_m - \delta_m^*)$, $\tilde{\xi}_i = \xi_i(\hat{x},\tau,\alpha_i,\delta_i) - \xi_i(\hat{x},\tau,\alpha_i^*,\delta_i^*)$ (for $i = 1,2,\cdots,m$),then the approximation error for the vector function $\Delta(x,\tau) = [\Delta_1(x,\tau),\cdots, \ \Delta_m(x,\tau)]^T$ can be expressed as

$$\hat{\Delta}(\hat{x},\tau \mid \Theta,\alpha,\delta) - \Delta(x,\tau) = (\Psi(\hat{x},\tau) - \alpha\Psi_\alpha(\hat{x},\tau) - \delta\Psi_\delta(\hat{x},\tau))\tilde{\Theta} + (\tilde{\alpha}\Psi_\alpha(\hat{x},\tau) + \tilde{\delta}\Psi_\delta(\hat{x},\tau))\Theta + w_1$$

where

$$\Psi(\hat{x},\tau) = \Psi(\hat{x},\tau,\alpha,\delta),\ \Psi_\alpha(\hat{x},\tau) = diag[\xi_{1\alpha_1}(\hat{x},\tau,\alpha_1,\delta_1),\cdots,\xi_{m\alpha_m}(\hat{x},\tau,\alpha_m,\delta_m)],$$

$$\Psi_\delta(\hat{x},\tau) = diag[\xi_{1\delta_1}(\hat{x},\tau,\alpha_1,\delta_1),\cdots,\xi_{m\delta_m}(\hat{x},\tau,\alpha_m,\delta_m)],\ \xi_{i\alpha_i}(\hat{x},\tau,\alpha_i,\delta_i)$$

and $\xi_{i\delta_i}(\hat{x},\tau,\alpha_i,\delta_i)$ denote partial derivatives of ξ_i with α_i and δ_i ,respectively. w_1 is the residual term.

Proof. Denote $(\tilde{\Delta}_i)_{m\times 1} = \hat{\Delta}(\hat{x},\tau \mid \Theta,\alpha,\delta) - \Delta(x,\tau)$

$$\tilde{\Delta}_i = \hat{\Delta}_i(\hat{x},\tau \mid \theta_i,\alpha_i,\delta_i) - \Delta_i(x,\tau) = \xi_i(\hat{x},\tau,\alpha_i,\delta_i)\tilde{\theta}_i + \tilde{\xi}_i\theta_i - \tilde{\xi}_i\theta_i + \tilde{\xi}_i\theta_i^* + \bar{\varepsilon}_i \tag{20}$$

where $\bar{\varepsilon}_i = \hat{\Delta}_i(\hat{x},\tau \mid \theta_i^*,\alpha_i^*,\delta_i^*) - \Delta_i(x,\tau)$.

Using the Taylor expansion of $\xi_i(\hat{x},\tau,\alpha_i,\delta_i)$ in (α_i^*,δ_i^*) ,we derive

$$\begin{aligned}\xi_i(\hat{x},\tau,\alpha_i,\delta_i) &= \xi_i(\hat{x},\tau,\alpha_i^*,\delta_i^*)+(\alpha_i-\alpha_i^*)\xi_{i\alpha_i}+(\delta_i-\delta_i^*)\xi_{i\delta_i}+o(\hat{x},\tau,\alpha_i-\alpha_i^*,\delta_i-\delta_i^*)\\ &= \xi_i(\hat{x},\tau,\alpha_i^*,\delta_i^*)+\tilde{\alpha}_i\xi_{i\alpha_i}+\tilde{\delta}_i\xi_{i\delta_i}+o(\hat{x},\tau,\tilde{\alpha}_i,\tilde{\delta}_i)\end{aligned} \tag{21}$$

where $o(\hat{x},\tau,\tilde{\alpha}_i,\tilde{\delta}_i)$ denotes the high order argument.

Substituting (21) into (20) yields

$$\tilde{\Delta}_i=(\xi_i(\hat{x},\tau,\alpha_i,\delta_i)-\alpha_i\xi_{i\alpha_i}-\delta_i\xi_{i\delta_i})\tilde{\theta}_i+(\tilde{\alpha}_i\xi_{i\alpha_i}+\tilde{\delta}_i\xi_{i\delta_i})\theta_i+(\alpha_i^*\xi_{i\alpha_i}+\delta_i^*\xi_{i\delta_i})\tilde{\theta}_i+o(\hat{x},\tau,\tilde{\alpha}_i,\tilde{\delta}_i)\theta_i^*+\bar{\varepsilon}_i$$

Denote $w_1=(w_{1i})_{m\times 1}$, $w_{1i}=(\alpha_i^*\xi_{i\alpha_i}+\delta_i^*\xi_{i\delta_i})\tilde{\theta}_i+o(\hat{x},\tau,\tilde{\alpha}_i,\tilde{\delta}_i)\theta_i^*+\bar{\varepsilon}_i$. We have

$$\hat{\Delta}(\hat{x},\tau\,|\,\Theta,\alpha,\delta)-\Delta(x,\tau)=(\Psi(\hat{x},\tau)-\alpha\Psi_\alpha(\hat{x},\tau)-\delta\Psi_\delta(\hat{x},\tau))\tilde{\Theta}+(\tilde{\alpha}\Psi_\alpha(\hat{x},\tau)+\tilde{\delta}\Psi_\delta(\hat{x},\tau))\Theta+w_1 .$$

The proof is completed.

Denote the residual term $w_1=[w_{11}^T\quad w_{12}^T]^T$, $\bar{w}_1=[0,(d_2-w_{11})^T,0,(d_4-w_{12})^T]^T$, $\bar{w}=[0\quad \bar{w}_1]^T$. By use of Theorem 2, (19) is rearranged as

$$\begin{aligned}\dot{\tilde{x}}(t)&=\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\bar{A}_{ij}\tilde{x}(t)+\sum_{l=1}^{r}\bar{A}_{il}\tilde{x}(t-\tau_l)]+\bar{w}\\ &\quad+\bar{B}[(\Psi(\hat{x},\tau)-\alpha\Psi_\alpha(\hat{x},\tau)-\delta\Psi_\delta(\hat{x},\tau))\tilde{\Theta}+(\tilde{\alpha}\Psi_\alpha(\hat{x},\tau)+\tilde{\delta}\Psi_\delta(\hat{x},\tau))\Theta]\end{aligned} \tag{22}$$

If the following parameter updating laws are chosen as

$$\dot{\Theta}=-\eta_1(\Psi(\hat{x},\tau)-\alpha\Psi_\alpha(\hat{x},\tau)-\delta\Psi_\delta(\hat{x},\tau))^T(\bar{B}^TP\tilde{x}) \tag{23.1}$$

$$\dot{\alpha}=-\eta_2\bar{B}^TP\tilde{x}(\Psi_\alpha(\hat{x},\tau)\Theta)^T \tag{23.2}$$

$$\dot{\delta}=-\eta_3\bar{B}^TP\tilde{x}(\Psi_\delta(\hat{x},\tau)\Theta)^T \tag{23.3}$$

where η_1 , η_2 and η_3 are positive constants. We have the following Theorem.

Theorem 3. For the nonlinear system (1), the control law is chosen as (15) composed of the fuzzy output feedback control law (6.3) and the compensator(16), (17) based on the adaptive time-delay fuzzy logic systems, and the parameter updating laws are chosen as (23), then the closed-loop system (19) satisfies the H_∞ performance

$$\begin{aligned}\int_0^T\tilde{x}^T(t)Q\tilde{x}(t)dt&\le\tilde{x}^T(0)P\tilde{x}(0)+\sum_{l=1}^{r}\int_{-\tau_l}^{0}\alpha_l\tilde{x}^T(v)\tilde{x}(v)dv+\frac{1}{\eta_1}\tilde{\Theta}^T(0)\tilde{\Theta}(0)\\ &\quad+\frac{1}{\eta_2}tr(\tilde{\alpha}^T(0)\tilde{\alpha}(0))+\frac{1}{\eta_3}tr(\tilde{\delta}^T(0)\tilde{\delta}(0))+\rho^2\int_0^T(\bar{w}^T\bar{w})dt\end{aligned} \tag{24}$$

Proof. Choose the *Lyapunov* function

$$V=\frac{1}{2}\tilde{x}^TP\tilde{x}+\frac{1}{2}\sum_{l=1}^{r}\int_{t-\tau_l}^{t}\alpha_l\tilde{x}^T(v)\tilde{x}(v)dv+\frac{1}{2\eta_1}\tilde{\Theta}^T\tilde{\Theta}+\frac{1}{2\eta_2}tr(\tilde{\alpha}^T\tilde{\alpha})+\frac{1}{2\eta_3}tr(\tilde{\delta}^T\tilde{\delta})$$

$$\begin{aligned}\dot{V}&=\frac{1}{2}\dot{\tilde{x}}^T(t)P\tilde{x}(t)+\frac{1}{2}\tilde{x}^T(t)P\dot{\tilde{x}}(t)+\frac{1}{2}\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t)\tilde{x}(t)-\frac{1}{2}\sum_{l=1}^{r}\alpha_l\tilde{x}^T(t-\tau_l)\tilde{x}(t-\tau_l)\\ &\quad+\frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}}+\frac{1}{\eta_2}tr(\tilde{\alpha}^T\dot{\tilde{\alpha}})+\frac{1}{\eta_3}tr(\tilde{\delta}^T\dot{\tilde{\delta}})\end{aligned}$$

From Theorem1, we have $\bar{A}_{ij}^TP+P\bar{A}_{ij}+\sum_{l=1}^{r}\alpha_l^{-1}P\bar{A}_{il}\bar{A}_{il}^TP+\sum_{l=1}^{r}\alpha_lI+\frac{1}{\rho^2}PP+Q<0$ for $i,j=1,2,\cdots,L$, therefore,

$\dot{V}\le-\frac{1}{2}\tilde{x}^T(t)Q\tilde{x}(t)+\frac{1}{2}\rho^2\bar{w}^T\bar{w}+[\tilde{x}^TP\bar{B}(\Psi(\hat{x},\tau)-\alpha\Psi_\alpha(\hat{x},\tau)-\delta\Psi_\delta(\hat{x},\tau))\tilde{\Theta}+\frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}}]+[\tilde{x}^TP\bar{B}\tilde{\alpha}\Psi_\alpha(\hat{x},\tau)\Theta$ $+\frac{1}{\eta_2}tr(\tilde{\alpha}^T\dot{\tilde{\alpha}})]+[\tilde{x}^TP\bar{B}\tilde{\delta}\Psi_\delta(\hat{x},\tau)\Theta+\frac{1}{\eta_3}tr(\tilde{\delta}^T\dot{\tilde{\delta}})]$. Using (23), we can yield $\dot{V}\le-\frac{1}{2}\tilde{x}^T(t)Q\tilde{x}(t)+\frac{1}{2}\rho^2\bar{w}^T\bar{w}$.

Integrating the above equation from $t=0$ to T yields (24). The proof is completed.

5. Simulation example

A 2-link manipulator system [8] is used to illustrate the effectiveness of the proposed method

$$\ddot{q}(t)+C(q,\dot{q})\dot{q}(t)+g(q)=B(q)\tau(t)+\sum_{i=1}^{r}\xi_i(t)q(t-\tau_i)+d'$$

where $C(q,\dot{q})=H^{-1}(q)C'(q,\dot{q})$, $g(q)=H^{-1}(q)g'(q), B(q)=H^{-1}(q)$, $d'=H^{-1}(q)d$, $q=[q_1,q_2]^T$, $\xi_i(t)$ is unknown and bounded, d is random noise with zero mean and variance 0.1, $r=2,\tau_1=0.1,\tau_2=0.2$.

By use of the proposed method in Theorem 3, the simulation results are shown in figure 1 and figure 2. The figure 1 shows the responses of the system states by only using the fuzzy output feedback controller. The figure 2 shows the responses of the system states under the fuzzy output feedback controller and the adaptive compensator.

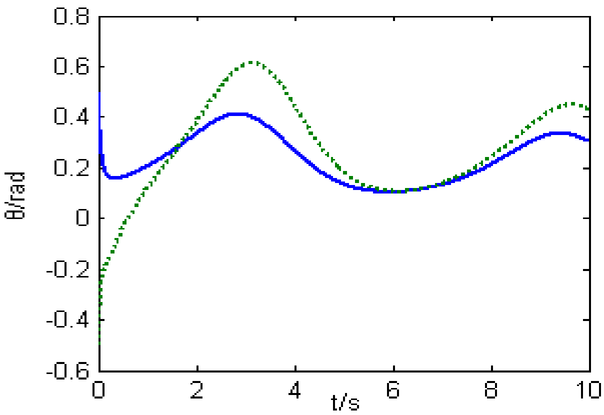

Fig. 1: The responses of the system states by only using fuzzy output feedback controller: angle q_1 (solid line), angle q_2 (dotted line) .

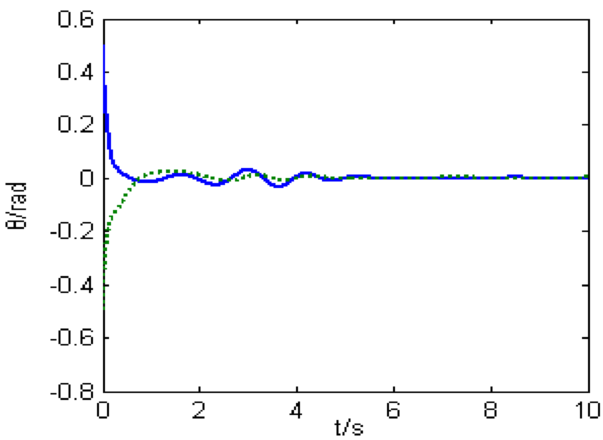

Fig. 2: The responses of the system states under fuzzy output feedback controller and adaptive compensator: angle q_1 (solid line),angle q_2 (dotted line) .

The simulation results demonstrate that the proposed control scheme can guarantee to stabilize the uncertain nonlinear time-delay system rapidly.

6. Conclusion

Combining both kinds of fuzzy logic forms including fuzzy T-S model and adaptive fuzzy logic systems, this paper presents a tracking control scheme for a class of uncertain nonlinear time-delay systems. The fuzzy T-S model is used to approximate the nonlinear systems, and the fuzzy control controller is designed to guarantee the stability of the fuzzy model. The modeling error and the uncertain nonlinearities are eliminated by a compensator based on the adaptive time-delay fuzzy logic systems. The simulation results demonstrate that the control scheme is effective.

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