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The numerical analysis on the model of human body - springboard system *

Yong Jiang [1, 2, +], Hongguang Bao [3], Youdao Shen [2]

[1] Dept. of Mathematics, Nanjing University, Nanjing 210093, China
[2] Dept. of Applied Mathematics, Nanjing University of Science and Technology, Nanjing 210094, China
[3] Nanjing First Hospital, Nanjing University of Medicine, Nanjing 210006, China

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Abstract. This paper describes the simulation for Human-spring board based on the model of Human-spring board which is a system of differential equations. Different control functions were employed to simulate relative human movements on spring board for diving.

Keywords: mathematical modelling, simulation, springboard diving, numerical analysis, biomathematics.

1. Introduction

Further to the statement of [1]-[3], we have two ways to solve out the movement $y(t)$ or $\theta(t)$ of the athlete for getting the best jumping. One of them is to find out the extremum of the function [4], another is to do computer simulations for obtaining the better function $y(t)$ or $\theta(t)$. Even for the former, we should know the class of function. Before we do that, we may use computer to do simulation for getting some of comprehension with regard to the class of functions. So it is important to do the latter. This just is the aim of this paper. In this paper, firstly, we consider how to construct the fundamental class of the control functions, then we will talk about the calculation of numerical difference and numerical integral on the analytic form of $y_c(t)$ and $y_{o_1}(t)$, finally, we show some results of simulation including the influence of control function to the output of the results and the influence of parameters to the output of the results. The relative analysis will be useful for guiding the training of athletes and selecting the fundamental class of control functions to find out the optimal solution.

2. Construction of the Control Function

For computer simulation, firstly we have to suppose the different classes of functions to instead of the control function.

There are several methods below to determine the control function $y(t)$ of the human-springboard. (1) Analyzing the practical record films of athlete movements by using the computer numerical analysis [5] to find out the variant curve/numerical discrete data of the mass center of human body and the movement curve of free end of springboard, then to fit out the control function [6]. (2) Assuming a kind of special function can be used instead of the control function, such as trigonometric functions, polynomial, etc. That means we try to suppose that the control function belongs to a kind of special function, then to find out the relevant parameters. (3) Improving the numerical curve by using the mouse. And also we can smooth the discrete data by using either curve fitting or polynomial fitting.

Some of the methods above will be used to construct the function class of the control function.

3. The Numerical Difference and its Smoothness

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+ Corresponding author. Tel.: +86-25-84315877.
E-mail address: math@mail.njust.edu.cn.

Based on the mathematical analysis, the differential coefficient of analytic function can be defined as

$$y'(t) = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}.$$

While the expression of function only can be expressed by discrete points, we can use the approximate method below to calculate differential coefficient of the function. That is

$$y'(t) \approx \frac{y(t+\Delta t) - y(t)}{\Delta t},\ y'(t) \approx \frac{y(t) - y(t-\Delta t)}{\Delta t},\ y'(t) \approx \frac{y(t+\Delta t) - y(t-\Delta t)}{2\Delta t}.$$

For getting a good precision, △t must be very small. But in practical calculation, $y(t+\Delta t)$ and $y(t)$ (or $y(t+\Delta t)$ and $y(t-\Delta t)$, $y(t)$ and $y(t-\Delta t)$) are very closed when △t is very small，so, the efficiency data will be lost after we subtract $y(t)$ from $y(t+\Delta t)$ (or subtract $y(t-\Delta t)$ from $y(t+\Delta t)$, subtract $y(t-\Delta t)$ from $y(t)$), in other words, that means that the precision will be lower. So we can think that this kind of method has no practical significant. Instead, we use the approach of curve fitting to solute out the differential coefficient of the control function, that is, we use the fitting polynomial $P_m(t)$ to approximate function $y(t)$, then we use the differential coefficient of the function $P_m(t)$ on a certain point to approximate the differential coefficient of the function $y(t)$ on that point.

Actually, we get the formulae below for the calculation of the difference.

$$Y'_{+0} = \frac{-Y_{+2} + 8Y_{+1} - 8Y_{-1} + Y_{-2}}{12\Delta t},\ Y''_{+0} = \frac{-Y_{+2} + 16Y_{+1} - 30Y_{+0} + 16Y_{-1} - Y_{-2}}{12(\Delta t)^2}.$$

And for starting two points, we have

$$Y'_0 = \frac{-21y_0 + 13y_{+1} + 17y_{+2} + 2y_{+3}}{12\Delta t},\ Y'_1 = \frac{-11y_0 + 3y_{+1} + 7y_{+2} + y_{+3}}{12\Delta t}.$$

For ending two points, we have similar formulae.

4. Numerical Integral

After getting the predictive control function and its differences, we can use the analytic form stated on the chapter 3 to get the numerical solution of $y_c(t)$ and $y'_c(t)$ by using the general numerical integral formula. Similarly, we can also obtain the numerical solution of $y_{o_1}(t)$ and $y'_{o_1}(t)$.

5. Numerical Simulation on the Model

Based on the above analytic expression and accordant computation, we develop an applied software named Human Body – Springboard to research the influence of different control function and parameters by using Borland C++ for Windows [7]. Then we have made series numerical imitative and artificial experiments. The result is fitted fare well with realities. Especially, we give relative analysis for guiding the training of the athletes and hint us to select the appropriate class of control function.

5.1. Influence of control function relative to output

We select different function instead of the input of control function to research the output of the system. Fundamental data of system parameters

m = 72.00, $\overline{m}$ = 45.00, K = 6543.31179, h = 0.67, T = 0.5129, y_{max} = 1.20, y_{min} = 0.49325,

where h is the height of the mass center of human body about previous jumping. Other parameters are the same as before.

Signification of output: Besides getting the numerical results, we can give the simulating proceeding by using graphical representation. Here we give relevant parameters and their signification.

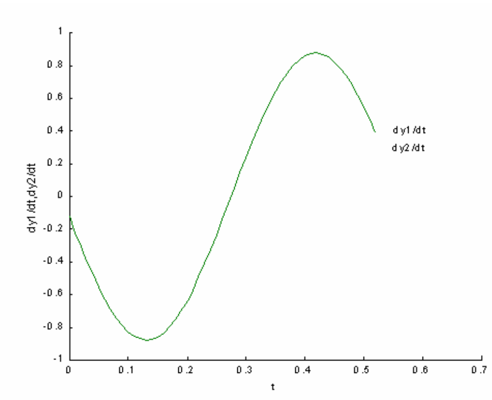
y_{cout} : The position of mass center of human body at time when human body take off the springboard, the unit is meter. $y_{o_1 out}$: The position of the free end of the springboard at time when human body take off the springboard, the unit is meter. V_{cout} : The velocity of mass center of the human body at time when the human body takes off springboard, the unit is meter/second. V_{o_1out} : The velocity of the free end of the springboard at time when human body takes off springboard, the unit is meter/second. H : The maximum of height of the human body after the human body takes off the springboard, the unit is meter. E_f : The value of function what is used to measure the effect of taking off, unit-less.

5.1.1. $y(t) = const = y_{\max}$

In this case, we assume $y(t)$ is constant. The initial values are

y_{cout} = 1.191710, V_{cout} = 0.389065, y_{o_1out} = -0.008290, V_{o_1out} = 0.389065, H = 0.007723, E_f = 0.000000.

The movement process of simulation is shown as following.



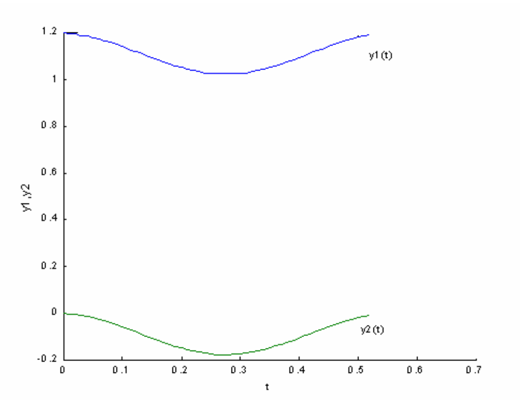


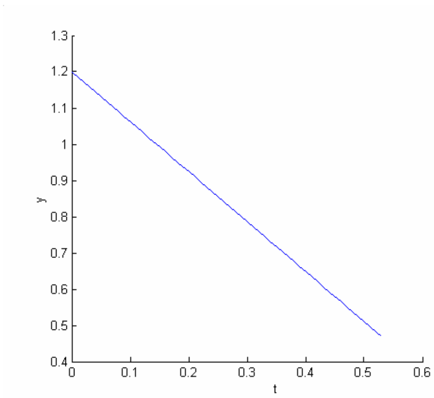
Fig.1. The variety curve of $\frac{dy_c}{dt}$ and $\frac{dy_{o_1}}{dt}$ when $y(t) = y_{\max}$

Fig.2. The variety curve of y_c and y_{o_1} when $y(t) = y_{\max}$

When control function $y(t) = y_{\max}$, we can find that V_{cout} and V_{o_1out} always are equal, the condition of taking off is not sufficiency, human body and springboard will not be separated. That means human body has same velocity with the springboard.

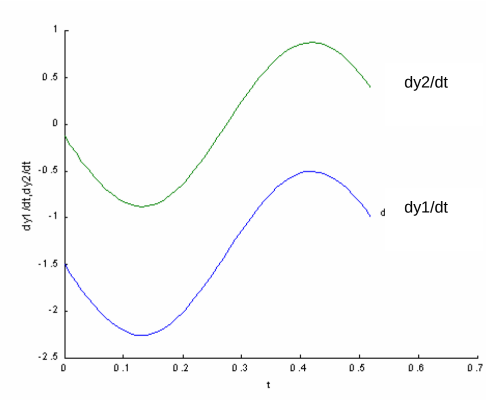
5.1.2. $y(t) = y_{\max} - \frac{y_{\max} - y_{\min}}{T} t$

It means that the control function is a linear function.



y_{cout} = 0.475176, V_{cout} = -0.988884, y_{o_1out} = -0.008290, V_{o_1out} = 0.389065, H = 0.049892, E_f = 0.000000.

In this case, the movement process of simulation is as the following.



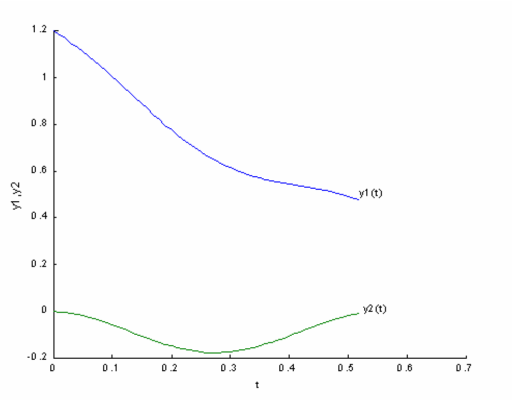
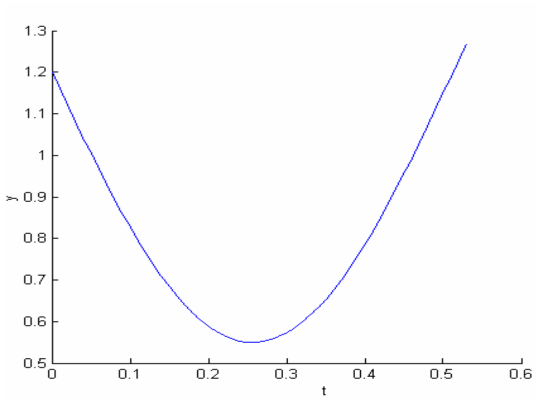


Fig.3. When $y(t) = y_{\max} - \frac{y_{\max} - y_{\min}}{T} t$, *curves of* $\frac{dy_c}{dt}$ *and* $\frac{dy_{o_1}}{dt}$

Fig.4. When $y(t) = y_{\max} - \frac{y_{\max} - y_{\min}}{T} t$, the curves of y_c and y_{o_1}

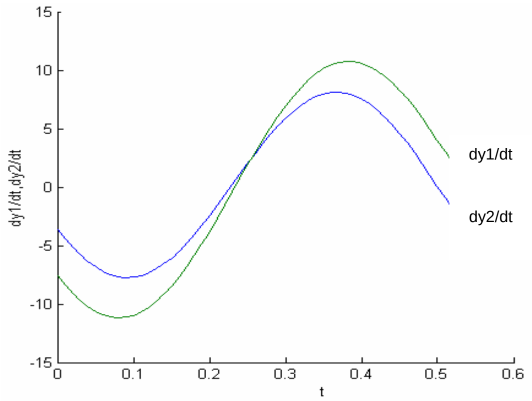
From here, we can find $V_{cout} < V_{o_1 out}$, so the condition of taking off the springboard is insufficient.

5.1.3. $y(t)$ is a sine function which period is 2*T*.



y_{cout} = 0.828520, V_{cout} = -1.955376, $y_{o_1 out}$ = -0.399738, $V_{o_1 out}$ = 2.022211, H = 1.023597, E_f = 0.314838.

The processing of simulation is as the following



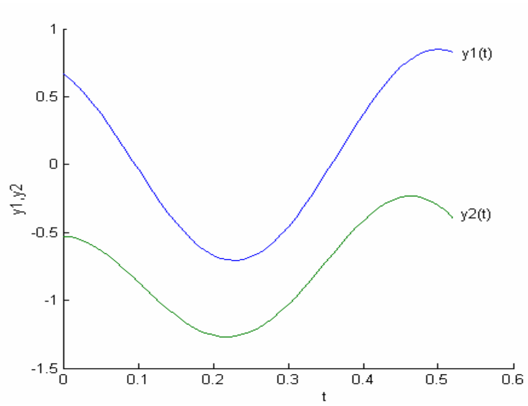
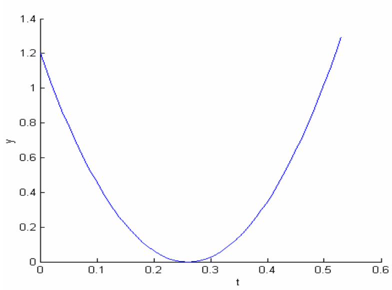


Fig.5. The curves of dy_c / dt and dy_{o_1} / dt when the control function is sine function

Fig.6. The curves of y_c & y_{o_1} when the control function is sine function

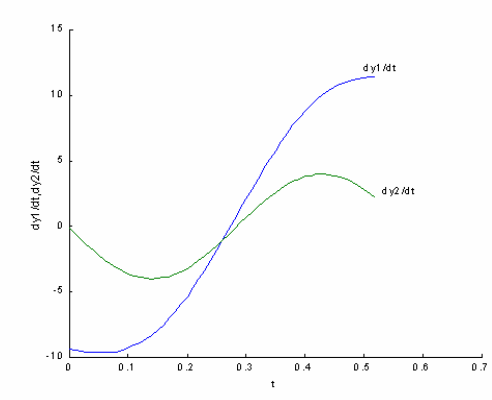
Now, $V_{cout} > V_{o_1 out}$, the condition of taking off the springboard is also sufficient.

5.1.4. Using arbitrary determined curve as the control function, showing as the figure below:



y_{cout} = 1.136187, V_{cout} = 11.398283, $y_{o_1 out}$ = -0.063813, $V_{o_1 out}$ = 2.167514, H = 6.628615, E_f = 0.991390.

The process of simulation is as the following:



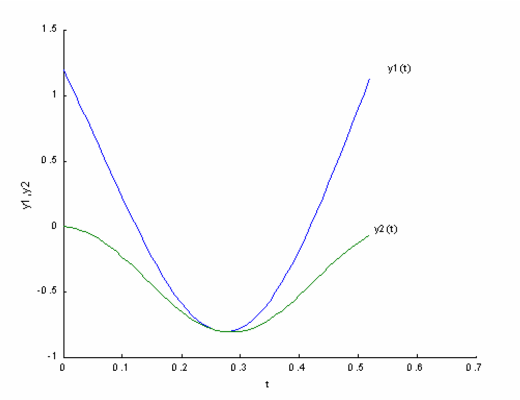


Fig.7. The curves of $\frac{dy_c}{dt}$ and $\frac{dy_{o_1}}{dt}$ when the control function is an arbitrary determined function.

Fig.8. The curves of y_c and y_{o_1} when the control function is an arbitrary determined curve

Further to the discussion above, we can find that the control function $y(t)$ can be selected. But it has no practical signification if it does not suffice to physical limitation. So we know:

The simulation software designed by this paper is hale. It can get the result which sizes up to practical output if the input date is accord with real those.

The control function should be collected to fit the physical case of the human body. It is matching the practical condition very well.

The control function will directly influence the height of jumping. Relatively, the treading control of athlete in the processing springboard diving will directly influence the final height of jumping.

Usually, the movement of the human body follows the fundamental movement mechanics under Newton movement laws, from the above simulation also we find that the better function curve of control function *f*(*t*) is approximate to ‘U’, it hint us to choose the polynomial (under 3 degree) as relevant class of the control function.

5.2. The Influence of Parameter to The System

By the assumptions of the model, we have $y(t) = 2L\cos\theta(t)$, $\theta_{\min} \le \theta(t) \le \theta_{\max}$. So, we can see, $y(t)$ is a trigonometric function which period can be changed. For the convenience of discussion, in this paper, we might as well let that $y(t)$ is a fixed period sine function, which period is 2T. Now we let $y(t) = y_{\max} - f\sin\frac{\pi}{T}t$, hence $y'(t) = -\frac{f\pi}{T}\cos\frac{\pi}{T}t$, and $y'(0) = V_0 = -\frac{f\pi}{T}$. So

$$y(t) = y_{\max} + \frac{V_0 T}{\pi}\sin\frac{\pi}{T}t.$$

And also from the initial condition here $V_0 = V_{10} - V_{20}$, $\frac{1}{2}mV_{10}^2 + \frac{1}{2}\tilde{m}V_{20}^2 = mgh$. It is easy to obtain the range of V_0 as $0 \geq V_0 \geq -\sqrt{2gh}$.

Now, we consider several aspects about the influence of parameters to the system respectively.

1. The influence of relative initial velocity V_0 of human body and the springboard for the system

We give parameters as we used before. By calculating, we can get - 3.62381 $\leqslant V_0 \leqslant$ 0. To limit the range as stated above, we give different value of V_0 , the results of calculation are given in the table 1.

Table 1. The influence of the relative velocity V_0 between human body and the springboard to the system.

V_0	y_{cout}	y_{o_1out}	V_{cout}	V_{o_1out}	H	E_f
-3.62	0.666255	-0.559439	4.199846	0.583269	1.566189	0.381402
-3.50	0.668817	-0.556025	4.102675	0.583269	1.527590	0.373896
-3.00	0.679489	-0.541804	3.697794	0.700630	1.377126	0.341331
-2.50	0.690164	-0.527581	3.292925	0.795288	1.243396	0.306890
-2.00	0.700830	-0.513358	2.888042	0.889933	1.126388	0.271035
-1.50	0.711511	-0.499136	2.483170	0.984589	1.026109	0.234581
-1.00	0.722185	-0.484913	2.078294	1.079239	0.942557	0.198804
-0.50	0.732858	-0.470691	1.673416	1.173888	0.875731	0.165560
-0.25	0.738194	-0.463580	1.470977	1.221213	0.848591	0.150648

From the results listed in the table 1, we can find that there is a certain relationship between the jumping height *H* of human body and -V_0 . That is, the bigger -V_0 is, the higher the height of jumping get. Due to $-V_0 = V_{20} - V_{10}$, and that the velocity of springboard is very small at the moment of human contacting with springboard, the velocity of springboard is approximate to zero, that is, the bigger $-V_{10}$ is, the higher *H* is. Due to that -V_{10} can be consider that it depends on the height of previous jumping, so in the practical springboard diving processing, the higher the previous height of jumping is, the higher the height of next jumping is. It fit to the practical instance very well. In another hand, while the athlete just contacts with the springboard, the performance which the athlete adopts is also a key to influence the value of *H*. That means, it is very important that the athlete distribute reasonably velocity V_0 . Generally, after human body whereabouts, the athlete should not tread rapidly while human body just contact with the springboard, instead of that, just lets the body to go down continually freely with the springboard.

2. The influence of the rigidity of the springboard for the system

Let V_0 = -3.50, other parameters are the same as those above, give different values of *K*, we get the relative results, which is listed in the following Table 2.

Table 2. The influence of rigidity of springboard to the system

K	y_{cout}	y_{o_1out}	V_{cout}	V_{o_1out}	H	E_f
6543	0.668779	-0.5566063	4.103804	0.607113	1.528024	0.373992
6500	0.663386	-0.561456	4.257568	0.760877	1.588227	0.386745
6000	0.575348	-0.649494	5.965240	2.468549	2.390863	0.483790
5500	0.436998	-0.787844	7.460978	3.964287	3.277110	0.507599
5000	0.243420	-0.981422	8.647850	5.151159	4.058997	0.485023
4500	-0.009565	-1.234407	9.424520	5.927830	4.522149	0.430076
4000	-0.325091	-1.549933	9.688251	6.191560	4.463797	0.348977
3000	-1.171927	-2.397648	8.345682	4.725298	2.381665	0.121491
2000	-2.251035	-3.476756	3.760318	0.139934	-1.529607	0.000000
1000	-3.519346	-4.745067	-4.315922	-7.936306	-2.568979	0.000000

where *K* expresses the rigidity of the springboard. From the above table we can realise that the rigidity of the springboard will influence directly the height of jumping and the rate of using energy. The springboard diving mainly makes human body to get the height as high as possible by using the elasticity of the springboard. So, within the certain bound, the smaller *K* is, the smaller the rigidity of springboard is. It is a soft board so called, which can get higher height. But when *K* goes little over the certain bound, *H* will decrease, which tells us that if the board is too soft it is not advance for jumping. By the way, we can know, the bigger the rigidity of springboard is, the smaller the after-shake is, the less the losing of energy is, so the high the rate of using energy is. Meanwhile, we found, it dose not always have higher rate of using energy, even if the height of jumping. Some time it can get higher height, but the after-shake is very big. In this case, the rate of using energy is lower. It matches to the practical case of springboard diving very well.

3. The influence of *h* of the first time jumping to system

Let V_0 = -3.50, other parameters are the same as those above. We give different values of *h*, then get the relative results, which are shown in the following Table 3.

From these we can see, there are apparent relation ship between the height of first jumping and the height of the second jumping. The higher the height of the first jumping is, the more useful it is to help to get the second jumping done. So in the progressing of practical spring-diving, after the athlete walking springboard, while he/she will firstly step-jumping and then jumping by treading springboard, he/she should make the height as he/she can. The problem is how to get the highest height of the first time jumping? Commonly, it depends on the violent strength of the athlete on moment. The stronger the violent strength of athlete is, the higher the height of jumping is.

Table 3 The influence of the first jumping height h to the system

H	y_{cout}	y_{o_1out}	V_{cout}	V_{o_1out}	H	E_f
0.00	-0.090040	-1.314882	3.325048	-0.171642	0.474039	0.055830
0.25	0.242184	-0.982658	2.904932	-0.591758	0.672726	0.086227
0.50	0.500546	-0.724296	3.553983	0.057293	1.144974	0.209582
0.75	0.746757	-0.478085	4.378947	0.882256	1.725082	0.477907
1.00	0.986676	-0.238166	5.294944	1.798253	2.417106	0.832252
1.25	1.222576	-0.002267	6.269160	-0.002267	3.227798	0.973520
1.50	1.455618	0.230776	7.284690	3.787999	4.163104	0.884627
1.75	1.686499	0.461657	8.331553	4.834862	5.228069	0.751234
2.00	1.915668	0.690826	9.403207	5.906516	6.426908	0.643092

Because the varieties of value will directly influence the varieties of V_0, we must consider the influence of V_0 to *h*, and try to let V_0 as big as possible. Let $V_0 = \max\{-\sqrt{2gh}, \frac{\pi}{T}(y_{\min} - y_{\max})\}$.

Then, by giving different *h* we can find out the result in the table 4. From the table 4, we can know that the value of V_0 changes with *h*. And *H* increases as the previous height of jumping increases. Meanwhile, we can also see, the rate using energy can reach maximum when *h* locate at certain range. That means that it does not always increase with *H*. So, in the progressing of spring-diving, it is not always good with higher height of first time jumping. Relatively, the higher height of the first time jumping is advance to control V_0, it can give wider range of varieties of V_0, so that it is advance to give enough time and space for adjusting movement by himself / herself.

Based on the discussion above, for getting the height of jumping as high as possible in the processing of springboard diving and the rate of using energy as high as possible, the athlete should try to get the height of the first time jumping as high as possible. That means that the athlete should have strong violent strength of treading the springboard.

While the human body just contact with the springboard as the whereabouts of the human body, at first do not strongly tread springboard. At first the human body goes down to freely contact with springboard. This can decrease the absolute value of V_{20}, so that it can increase absolute value of V_0, and then get the height as high as possible.

Table 4. The influence of h to the system when considering V_0

H	V_0	y_{cout}	y_{o_1out}	V_{cout}	V_{o_1out}	H	E_f
0.00	0.000000	-0.015326	-1.215326	0.490913	0.490913	-0.003030	0.000000
0.25	-2.213594	0.269644	-0.946068	1.863254	-0.348247	0.446772	0.045259
0.50	-3.130495	0.508433	-0.713786	3.254776	0.127241	1.048922	0.188543
0.75	-3.834058	0.739624	-0.487589	4.649441	0.819008	1.842548	0.496707
1.00	-4.328945	0.968980	-0.261746	5.966198	1.641337	2.785978	0.841387
1.25	-4.328954	1.204881	-0.025845	6.940407	2.615546	3.662496	0.979315
1.50	-4.328954	1.437924	0.207198	7.955932	3.631071	4.667356	0.915778
1.75	-4.328954	1.668807	0.438081	1.668807	4.677946	5.804039	0.795381
2.00	-4.328954	1.897972	0.667246	10.074457	5.749596	7.076272	0.688600
2.25	-4.328954	2.125740	0.895014	11.166353	6.841492	8.487344	0.606804
2.50	-4.328954	2.352335	1.121609	12.275239	7.950377	10.040167	0.545781
2.75	-4.328954	2.577932	1.347206	13.398587	9.073726	11.737226	0.499883

Controlling the position of mass centre of human body is necessary. It means that human body should maintain the steadfastness on the horizontal direction, so that the movement comportment is stable, the springboard will not be shaken, it can increase the rate of using energy.

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