

## Model study on loss assessment of invasion species\*

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**Abstract:** By taking the *Eupatorium adenophorum*, the paper systematically analyzes the influencing factors and theoretical system of loss assessment on the economy and environment in invasive regions caused by invasive species, especially plant species, which its loss assessment depends on its diffusion area in adaptable living regions and impact measure of losses. The paper discusses the Verhulst—Pearl equation satisfied by the ratios of invasive species diffusion area in its adaptable living region; by using the niche theory, given calculated formulas about temporal niche, horizontal niche and vertical niche of species in invasive ecosystem before and after of biological invasions; by using the value which niche values of species after biological invasions minus its niche values before biological invasion, the paper established the impact measure of losses led by biological invasions species. Based on the above, the paper builds a theoretical model of direct loss intensity function and indirect loss intensity function, and get loss function models about biological invasions, gives corresponding sensitivity analysis and theoretical research of those models, which those theories and methods extensively adapt for all invasive species such as animal species and plant species.

**Key words:** biological invasions, living region, impact measure, niche, loss function of unit area, mathematical model

Biological invasions occur when organisms are transported by contrived or natural ways to a new district, where their descendants form groups, proliferate, and spread quickly [1], this caused great loss on social economy, environment and people's health. It had been a problem in ancient ages. With the development of human civilization and the improvement of human transportation facilities, the flow of material and people between nations and areas is becoming more and more frequent so that more contrived spreading ways are available for the biological invasions. In the past 200 years, the number of invasive species had increased by several orders of magnitude [2]. Though it is a small probability event that the foreign species finally became the invasive species [3], once it happened, the losses and controlling costs will be egregious. According to the calculation, the loss on economy and environment caused by the invasive species is 137 billions dollars per annum only in the USA [4]. In China, by glancing at statistics, the loss on economy caused by the major invasive species is up to 57.4 billion RMB per annum, and the total loss will be several hundreds billion RMB. Obviously, the study on the invasive species has become a major environment problem we have to face.

The kernel problem of the biological invasions study is the assessing method and theoretical system to calculate the loss caused by the invasive specie. Until now, there is not a specialized, complete theoretical system of loss assessment on the economy and environment caused by invasive species. Every nation has its own evaluating method, even in the same country, the losses obtained by different expert groups is differential, but the method can be divided into objective assessment and subjective assessment, also there are studies on models, like Community Assembly Model [6], Null Model [7] and Multiple Regression Model, etc. But the study on the influence assessment of biological invasions is still weak, there is not a unified understanding to the definition and the assessment method of the influence caused by the foreign species.

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This study tries to give a complete theoretical system of loss assessment on the economy and environment in invasive region caused by invasive species, establishes an extensive and related mathematical model and assessment method by the malignancy weed-Eupatorium adenophorum.

## 1. Structure system of the loss assessment caused by the biological invasions

A biological invasion is a complicated process and it is relevant with the biological characteristic of the invasive species and the environment of the invasive regions. Successful invasive species, which can cause great influence to economy and environment, should have the basic characteristic as follows [5]:

(1). The invasive group should have a certain quantity, and its space distribution should be high density and large scale; (2). The invasive group should have strong potentials of propagating and spreading, but low death ratio; (3). The invasive group should have a certain ability of anti- interference and strong flexibility to the environment; (4). An available region that can be invaded.

Because the invasive species have different living history and evolution process from the native species, they have no natural enemy and their flexibility is high, once they succeed, their group number will enlarge and explore adequately and cause disastrous damages to the ecosystem.

A common method to measure the influence of invasive species is to calculate the loss on economy and environment caused by invasive species or the economic expenses used to eliminate or control the invasive species [11]. The loss on the economy caused by invasive specie can be divided into direct ones and indirect ones according to the values of usability and the values of unusability of environment. The direct loss caused by biological invasions is the loss on the economy and damages caused by the dynamic changes of native species groups due to the biological invasions. Correspondingly, after the biological invasions, the indirect loss on the economy includes all losses which are to destroy the integrity of structure and functions of the original ecosystem, to weak the fastness, restorability, stability and sustainability and to cause further negative influence to the national economy. The ecosystem can be scaled by three characteristics indexes, namely the structure (organization), function (livingness), and the flexibility (elasticity), the relevant loss caused by invasive species can also be scaled by the structure, function and flexibility. Now we study the influence of ecosystem caused by invasive species mainly concentrating with four ecosystems: forest, grassland, farmland and everglade, the classify system and measure index of indirect loss is given by the 21 items of the four ecosystems [12, 13]. Now we call the native species that are very important to national economy as the key species.

We assess the loss that caused by biological invasions mainly by two steps as follows:

- (1). Confirming the key species and the kind of ecosystem in invasive region.
- (2). Influence measures on the key species and the ecosystem caused by invasive species.

We may confirm the key species in invasive regions through investigation. However, it is difficult to confirm the ecosystem influenced by invasive species. But we only need to consider the main influence of the invasive species specifically. Taking the Eupatorium adenophorum for example, it is a malignancy weed in the world generally accepted, which origin region is Middle America. The Eupatorium adenophorum was transported from Thailand and Burma into Yunnan in 1940s, it spreaded to north quite fast, now it is distributing widely in Yunnan, Guangxi, Guizhou and Sichuan, even in Hubei some straggly distributions. According to the investigation about the damage of Eupatorium adenophorum in China [14-15], it distributes in roadside, slope, drain-side, grassland side and farmland, seizes the grass field and forest with crown density of lower than 70% (such as economic forest, teagarden and fruit garden). For rotate fields, wasteland and farmland systems, especially in rotate fields and wasteland, the Eupatorium adenophorum can establish its group very quickly and form a simplex superior group, but it can not do harm to a whole forest. So, we only need to consider the grassland ecosystem, farmland ecosystem, and economic forest when we assess the loss caused by the invasive species-Eupatorium adenophorum.

There are 3 factors when measuring the influence of the invasive species: the distributing area, the density of invasive species and the impact measure [6]. Directly, it is reasonable to take the density of invasive species to be the measure of the invasive influence in invasive regions. Because any biomass (space or energy) occupied by the invasive species can not be used by competitor at the same time. The change of the density of invasive species in the same region is often used to be as the measure of the influence caused by invasive species, but the density does not represent the influence of invasive species when comparing with the different invasive species. We can consider this situation in two aspects, firstly, higher density of

invasive species caused bigger invasive strength and fast spreading speed, and these will be shown in the acreage increasing; Secondly, the density and impact measure of invasive species can be united and we may consider the loss in the measure of biodiversity. So, we study the influence measure of invasive species according to the spreading and loss effect of the invasive species in the living area.

## 2. The influence measure of invasive species

### 2.1. Spreading model of the invasive species' living area

One basic characteristic of the invasive species is the spreading speed (or the acreage of invasive region) of the invasive groups in geographical distributing region. The earliest model of invasive speed was a reaction-spread model established according to the random changing probability density [17], such as the Integro-difference Equation Model [16] and the Differential Equation- matrix Model [18]. But all of these are over simplified that they can only describe the spreading speed of signal invasive species in a small area. Actually, it is difficult to predict the invasive speed of biological invasions.

The investigation of the spreading situation of *Eupatorium adenophorum* in China obtained by Chinese experts shows the *Eupatorium adenophorum* has different spreading speed in different areas. Take the Liangshan (Sichuan Province) for example, since the invasion of *Eupatorium adenophorum* first detected in 1978, the invasive area increased from 280hm<sup>2</sup> (1984) to 537720hm<sup>2</sup> (2001). It had enlarged 192,000% in 17 years, nearly satisfies the exponential increase, and it spreads towards southeast 30km per annum. Yunnan, is the earliest province invaded by the *Eupatorium adenophorum*, which spreads towards north 10km per annum, the invasive area was 11,000,000hm<sup>2</sup> in 1983 and became 30,000,000hm<sup>2</sup> in 2001, the invasive area had expanded nearly 300% in 18 years. The reason for the different increase of invasive area (or spreading speed) is complicated. According to the internal investigative literatures [19-20], the *Eupatorium adenophorum* seed spreads into new region mainly by wind, water and vehicles, *Eupatorium adenophorum* enjoys warmth and light, it is relevant with geographical (elevation, slope, the distance to the headwaters, etc.), climate (average air temperature of year, average annual rain fall, number of days that without frost, relative humidity, etc.) and environment (group structure, soil richness) in the invasive region. So it is easy to explain the difference of increasing invasive area between Liangshan and Yunnan caused by the different ecosystem environment and geographical positions: the former has greater quantities of grass field that is invaded easily (note: it has 22.68% of the whole state area), the ecosystem has become flimsy because of the long-term excessive development; and the 108 National road through the whole area while the east faces the Jinsha River; all of these caused the invasive area (spreading speed) increasing quite fast. Another main reason of the different invasive level is, because Liangshan south faces Zhaotong (Yunnan) and Chuxiong (Yunnan), southwest faces Pazhuhua (Sichuan), all these areas are invaded by *Eupatorium adenophorum* seriously.

There are many factors that can influence the spreading speed of *Eupatorium adenophorum*. We may study by the increasing invasive areas in a particular district. According to the studies of preventing *Eupatorium adenophorum* world wide, none of them has any perfect effect and cannot control the crazy spreading speed of *Eupatorium adenophorum*. We suppose the spread of *Eupatorium adenophorum* in China is still completely under natural situation.

Suppose the proportion of invasive areas taken in the invasive region by the invasive species at the time  $t$  is  $S(t)$ . There is a direct ratio between the increase of  $S(t)$ ,  $S$  and its function  $f(S)$ , where  $f(S)$  is the retardant factor of  $S$  by the environment. Because the bigger proportion of invasive area is taken, the control effect on the further increasing is more effective. We suppose  $f(S)$  to be the linear form of  $S$ , namely:  $f(S) = k[1 - S(t)]$ . We mark the proportion of invasive area to be  $S_0$  at a certain begin time ( $t = t_0$ ), and  $S(t)$  satisfies the famous Verhulst—Pearl retardant equation:

$$\begin{cases} \frac{dS}{dt} = kS(t)[1 - S(t)] \\ S(t_0) = S_0 \end{cases} \quad (2.1)$$

Here  $k$  is the average natural increasing rate,  $k$  depends on the biological characteristic of the invasive species, normally it remain a constant in the early invasive period. The theoretical calculation is

$$S(t) = 1/[1 + (1/S_0 - 1) \cdot e^{-k(t-t_0)}] \quad (2.2)$$

Discrete of model (2.1):  $S_t$  represents the invasive area taken in the invasive region at the time  $t$ , and we mark:  $(1/S_0 - 1) \cdot e^{kt_0} = C$ ,  $\lambda = e^k$  —limited rate of natural increase.

So: the difference equation on discrete time of (2.1) will be

$$\begin{cases} S_{t+1} = \frac{\lambda S_t}{1 + (\lambda - 1)S_t} \\ S_0 (knew), (t = 0, 1, 2, \dots) \end{cases} \quad (2.3)$$

$$k = \frac{S_{t+1} - S_t}{S_t} + \frac{S_{t+1} - S_t}{1 - S_t} \quad (2.4)$$

and the divergingly calculate formula can be obtained by (2.4) and (2.2).

## 2.2. Impact measuring model of the influence caused by invasive species

The influence on human beings caused by invasive species can be measured from five levels: the individual, descendibility, group trend, community and the influence of ecosystem [21]. The problem is how to describe the measure index of these influence level and assess the loss of unit area caused by the invasive species. In recent years, Peterson and Vieglais had given out the Ecological Niche Modeling [22, 23] based on the forecast of biodiversity. The model gives the forecast that the region or ecosystem may be invaded and it based on the study of the basic niche of invasive species. The conception of niche is first given by Grinnell: Niche is an ultimate distribution unit that is taken by species or subspecies. Here the ultimate distribution unit means the basic ecotype and space which living beings need to live provided by the environment. There are many factors that can influence the plant growth, such as ecology and climate, etc. However, for the invasive species, these comprehensive influences of all kinds factor finally represent the space distributing through the resource competitions. According to Gauss's competition exclusion principle and study [24], species with the same niche cannot live together for a long period. These researches give us many revelations: the many kinds of influences to human beings caused by invasive species mainly represent the resource competition between invasive species and native species. Once the invasion is successful, the loss on the economy and environment caused by the invasive species should be represent in the ecosystem, while the niche of the invasive species and native species are the representation of the comprehensive influences by the comprehensive factors. So, we may study the influence level of invasive species by the niche.

We determine the niche by the niche width determining formula that is calculated from the biodiversity information [22, 25]:

$$v_i = -(\sum_{k=1}^m p_{ik} \ln p_{ik}) / \ln m \quad (2.5)$$

Here  $p_{ik}$  is the proportion of specie  $i$  taken  $k=1$  in state  $k$  of a certain resource,  $m$  is the total amount of resource. We assume when  $p_{ik} = 0$ ,  $p_{ik} \ln p_{ik} = 0$  (because  $\lim_{p_{ik} \rightarrow 0^+} p_{ik} \ln p_{ik} = 0$ ). Normally, the bigger the niche width is, the stronger the competing power to resource the specie gets; we can use this to describe the activity range and degree of the specie.

Suppose the invasive specie  $N_0$  (such as *Eupatorium adenophorum*), the key specie under the influence of ecosystem in the invasive region is  $N_1$ , pre-invading state,  $L = 0$ , invading state,  $L = 1$ , other species are  $N_2, N_3, \dots, N_n$ . We start with the time niche, level niche and vertical niche.

(1) Setting the time niche state and index: divide the time into several periods according to the life of key species, such as dividing the time into several time stages  $T$  ( $T = 1, 2, \dots, m$ ) as the time resource sequence; take the proportion of a special number in unit acreage ( $1 \text{ m}^2$ ) as the index.

(2) Setting the level niche state and index: take several samples of several regions and form the resource sequence, take the proportion of fresh weight in unit acreage ( $1 \text{ m}^2$ ) as the index, which the fresh weight ( $\text{g/m}^2$ ) represents the result of the species competed in the community, such as the nutrient.

(3) Setting the vertical niche state and index: divide the units by the stem length of key species. Such as take the 1/3 of stem length to divide the four periods:  $[0,1/3)$ ,  $[1/3,2/3)$ ,  $[2/3,1)$ ,  $[1,+\infty)$  as the resource sequence of vertical niche state, here  $k = 1,2,3,4$  represent separately for the four states. Take the percent of hill number in unit acreage ( $1 \text{ m}^2$ ) under a certain situation to be the index. Pay attention: if the stem length is 1, it represents that the specie has occupied all the blow states. Actually, to some extent, the vertical niche state reflects the degree of the light competition in community.

We mark  $v_{ij}^{(L)}$  as the niche  $j$  of specie  $i$  in the state  $L$  and mark  $P_{ij}^{(L)}(T)$  as the proportion of specie  $i$  in the state  $L$  taken in the  $T$  time states of resource level  $j$ . Here  $j = 1,2,3$  represent separately for the time niche, level niche and vertical niche,  $P_{i3k}^{(L)}(T)$  is the proportion of specie  $i$  in the state  $L$  taken in the  $T$  time states at vertical resource level of height period  $k$ ; when  $L = 0$ , because without the invasive specie  $N_0$ ,  $i = 1,2,\dots,n$ ; when  $L = 1$ ,  $i = 0,1,2,\dots,n$ , here  $P_{i3}^{(L)}(T)$  can be calculated by the formula as follows:

$$P_{i3}^{(L)}(T) = \frac{-\left(\sum_{k=1}^4 P_{i3k}^{(L)}(T) \cdot \ln P_{i3k}^{(L)}(T)\right) / \ln 4}{-\left(\sum_{i,k} P_{i3k}^{(L)}(T) \cdot \ln P_{i3k}^{(L)}(T)\right) / \ln 4} = \frac{\sum_{k=1}^4 P_{i3k}^{(L)}(T) \cdot \ln P_{i3k}^{(L)}(T)}{\sum_{i,k} P_{i3k}^{(L)}(T) \cdot \ln P_{i3k}^{(L)}(T)} \quad (2.6)$$

According to the formula (2.5) we may get the calculating formula of niche width:

$$v_{ij}^{(L)} = -\left(\sum_{T=1}^m P_{ij}^{(L)}(T) \cdot \ln P_{ij}^{(L)}(T)\right) / \ln m \quad (2.7)$$

when  $L = 0$ ,  $i = 1,2,\dots,n$ ; when  $L = 1$ ,  $i = 0,1,2,\dots,n$ ;  $j = 1,2,3$ . We use vectors to represent the niche vectors of specie  $i$  before and after the Eupatorium adenophorum invasion happens, the  $v_i^{(L)}$ :

$$v_i^{(0)} = [v_{i1}^{(0)}, v_{i2}^{(0)}, v_{i3}^{(0)}] \quad (i = 1,2,\dots,n), v_i^{(1)} = [v_{i1}^{(1)}, v_{i2}^{(1)}, v_{i3}^{(1)}] \quad (i = 0,1,2,\dots,n)$$

So we may select different regions with different species in different ecosystems, obtain the data above and do relevant deals. According to the fact that Eupatorium adenophorum becomes the signal superior group in rotate field and wasteland in the 3rd year. This happens in grassland ecosystem, too [12, 15, 16]. So when selecting the invasive region swatch, we can select the different regions which were invaded for less than 3 years (such as 2 years), 3~5 years and more than 5 years. The same time we should obtain both the outputs before and after the invasion happens.

Colligating these discussions above, seeing directly, the unit acreage output of the key species is determined by the practical niche width. So after the biological invasions, the direct loss degree on economy should depend on the niche change of the key species, and the loss on ecosystem environment should be relevant with the niche change  $\Delta v = (v_0^{(0)}, v_2^{(0)}, \dots, v_n^{(0)}) - (v_0^{(1)}, v_2^{(1)}, \dots, v_n^{(1)})$  of the species in ecosystem, here  $v_0^{(0)} = 0$ .

Because what we care about is the biomass influence of the key species that caused by the invasive species, the review range should only be about the key species whose output can be influenced evidently. At the same time we may see the influence level of niche width from  $N_0$ . If considering the direct loss on economy and the indirect loss on environment brought by  $N_0$ , we may simplify the discussion of the niche width and niche overlap that are established above. Actually,  $P_{ij}^{(L)}(T)$  is a percentage, so we can calculate the percentage of the invasive species and other ones taken according to the data of key species. If we mark the invasive species and other ones as  $N_2$ , so we simplify the formulas (2.6) and (2.7), here

$$P_{1j}^{(L)}(T) + P_{2j}^{(L)}(T) = 1 \quad (j = 1,2,3; T = 1,2,\dots,m) \quad (2.8)$$

### 3. Establishing the loss function of biological invasions

#### 3.1. Loss level function

Loss level is the loss on unit acreage. Theoretically, the more the niche width changed, the bigger influence

to its output is caused. According to the analytic hierarchy process, we introduce a combined influence amount:

$$x_i = \sqrt[3]{|\Delta v_{i1} \cdot \Delta v_{i2} \cdot \Delta v_{i3}|} \quad (i = 0, 1, 2, \dots, n) \tag{3.1}$$

For each specie, the bigger the combined influence amount  $x$  is, the bigger the loss on species  $y = y(x)$  will be, especially, when  $x = 0$ ,  $y(0) = 0$ . When  $x = 1$ , namely  $\Delta v = (1, 1, 1)$ , this moment the specie dies out, the loss is  $y(1) = K \geq 1$ , here  $K$  lies on its proper value, and the relation between  $y$  and  $x$  is not linear relationship. To obtain the relation between loss amount  $y$  and the combined influence amount  $x$ , we must analyze the property it should satisfy first.

**3.1.1 Probability distribution of the combined influence amount**

If there is a set of division for the combined influence amount  $x$  of a certain specie, such as dividing  $[0, 1]$  into  $n$  periods, taking the intermediate value to be the typical value of the small period  $i$ . To a certain invasive region (or an invasive ecosystem), we suppose the relevant probability of  $x_i$  is  $p_i$ , we obtain the distribution law as follows:

X	$x_1$ $x_2$ ..... $x_n$
P	$p_1$ $p_2$ ..... $p_n$

We may simply mark it as  $\bigcup_{i=1}^n x_i \circ p_i \sim x_1 \circ p_1 \oplus x_2 \circ p_2 \oplus \dots \oplus x_n \circ p_n$ ,

where  $x_i \geq 0$ ,  $p_i \geq 0$ , and  $\sum_{i=1}^n p_i = 1$ . So the loss level and the combined influence amount distributions of a certain influenced specie are one to one correspondence.

**3.1.2 Establishing the loss function**

Because the bigger the combined influence amount  $x$  is, the bigger the loss on species  $y = y(x)$  will be, marking  $\succ$  as the preference relationship of the influence degree caused by invasive species to the native species, and the relationship should satisfies completeness, reflexivity and transitivity.  $\Gamma$  represents the circumstances group of the influence degree caused by invasive species to the native species. So there is a utility function (loss level function) [25] with preference relationship in  $\Gamma : y : \Gamma \rightarrow R$  satisfies:

$$\bigcup_{i=1}^n x_i \circ p_i \succ \bigcup_{i=1}^n y_i \circ p_i \Leftrightarrow y(\bigcup_{i=1}^n x_i \circ p_i) > y(\bigcup_{i=1}^n y_i \circ p_i)$$

where  $y_i = y(x_i)$ . The invasive species and the loss level of each species in ecosystem are one to one correspondence. It is easy to see the expectation of loss level has properties as follows [26]:

- (1)  $E(Y) = y(\bigcup_{i=1}^n x_i \circ p_i) = \sum_{i=1}^n y(x_i) \cdot p_i$
- (2)  $E(X) = \sum_{i=1}^n x_i \cdot p_i$ , all to the species that are influenced by the invasion:

$$y[E(X)] = y(\sum_{i=1}^n x_i \cdot p_i) < y(\bigcup_{i=1}^n x_i \circ p_i) = \sum_{i=1}^n y(x_i) \cdot p_i$$

We can also prove: when we neglect the increase affine transformation, the loss degree function is unique, namely  $\forall x_1, x_2 \in [0, 1], x_1 \neq x_2$ , we have  $y[(x_1 + x_2)/2] < [y(x_1) + y(x_2)]/2$ . Here  $y$  should be a down convex function.

Typical functions like exponential function and power function satisfy these relations, we suppose it is exponential function:

$y = a + b(1 - e^{\lambda x})$  here the parameter  $\lambda > 0$ , according to the conditions  $y(0) = 0$ ,  $y(1) = K$  we know

$$a = 0, \quad b = K/(1 - e^{\lambda}), \quad y(x) = K(1 - e^{\lambda x})/(1 - e^{\lambda}), \tag{3.2}$$

the parameter  $\lambda$  can be confirmed by the niche combined amount  $x$  and unit loss amount  $\Delta y$ .

We can obtain the indirect loss level (environment loss function level) of the specie relevantly:

$$\bar{y}(x) = \bar{K}(1 - e^{-\lambda x}) / (1 - e^{-\lambda}) \tag{3.3}$$

where  $\bar{K}$  is the expected value of other values.

### 3.1.3 Sensitivity analysis of loss level

For the combined influence amount  $X$  of a certain specie in the invasive ecosystem, looking upon the appearance of the specie as event  $A$ , the appearance of other species as event  $\bar{A}$ , thus the event that whether the specie appears can be looked upon as 0-1 distribution, suppose the probability of event  $A$  appears is  $p$ , suppose the swatches of the combined influence amount  $X$  of certain specie are  $X_1, X_2, \dots, X_n$ , its

summation  $\sum_{i=1}^n X_i \sim B(n, p)$ , so:  $D(\bar{X}) = p(1 - p)/n$ , the average standard deviation:

$$\bar{\sigma} = \sqrt{D(\bar{X})} = \sqrt{p(1 - p)/n} \leq 1/(2\sqrt{n}).$$

The relevant expectation [26] of loss function level  $y = y(x)$  is  $E(Y) = py(x) + (1 - p)\bar{y}(1 - x)$ ,

its variance is  $D(Y) = p \cdot y^2(x) + (1 - p) \cdot \bar{y}^2(1 - x) - E^2(Y) = p(1 - p)[y(x) - \bar{y}(1 - x)]^2$ .

For the given swatches  $x_1, x_2, \dots, x_n$ , the relevant variance is  $D(\bar{Y}) = \frac{p(1 - p)}{n} [y(x) - \bar{y}(1 - x)]^2$ . So the

average standard deviation is  $\sqrt{D(\bar{Y})} \leq \frac{1}{2\sqrt{n}} |y(x) - \bar{y}(1 - x)|$ .

## 3.2. Establishing the loss function of biological invasions

Suppose  $A_0$  is the normal regions of invasive species in a certain region, the key native species that are influenced by  $A_0$  are  $N_1, N_2, \dots, N_n$ , the proportion they taken in the normal region are  $k_1, k_2, \dots, k_n$ . Here

$0 < k_i < 1$ , ( $i = 1, 2, \dots, n$ ) and  $0 < \sum_{i=1}^n k_i \leq 1$ . The direct loss level of each specie is  $y_i(x_i)$ , we may

obtain the indirect loss level  $\bar{y}_i(1 - x_i)$  from the formulas (3.8) and (4.3), and  $A_0 \cdot S(t) \cdot k_i$  is the invasive area taken by the key specie  $i$  at the time  $t$ , so we can get the direct economic loss amount and indirect economic loss amount:

$$C_{direct}(t) = \sum_{i=1}^n y_i(x_i) \cdot A_0 \cdot k_i \cdot S(t), \quad C_{indirect}(t) = \sum_{i=1}^n \bar{y}_i(1 - x_i) \cdot A_0 \cdot k_i \cdot S(t)$$

Accordingly the total loss on economy is

$$C(t) = C_{direct} + C_{indirect} = A_0 \cdot S(t) \sum_{i=1}^n k_i [y_i(x_i) + \bar{y}_i(1 - x_i)] \tag{3.4}$$

From formulas (2.4),(3.2),(3.3) we can obtained:

$$\begin{aligned} \Delta C(t) &= A_0 \cdot \Delta S(t) \sum_{i=1}^n k_i [y_i(x_i) + \bar{y}_i(1 - x_i)] + A_0 \cdot S(t) \sum_{i=1}^n k_i \left[ \frac{dy_i}{dx_i} + \frac{d\bar{y}_i}{dx_i} \right] \Delta x_i \\ \Rightarrow \frac{\Delta C(t)}{C(t)} &= k(1 - S) + \frac{\sum_{i=1}^n k_i [\lambda_i y_i(x_i) - \bar{\lambda}_i \bar{y}_i(1 - x_i) + \frac{\bar{\lambda}_i k_i}{1 - e^{-\lambda_i}} - \frac{\lambda_i k_i}{1 - e^{-\lambda_i}}]}{\sum_{i=1}^n k_i [y_i(x_i) + \bar{y}_i(1 - x_i)]} \end{aligned} \tag{3.5}$$

These formulas reflect the sensitivity of total loss, it is relevant with the proportion taken by the invasive species in the invasive normal region, increasing rate and each level of loss assessment, it is nearly perfect, the same time we can see its dynamic changes (relevant with  $t$ ).

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