ISSN 1746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 1 (2005) No. 1, *pp*. 12-18

A game of energy allocation played by a central government and two local governments *

Zhanwen Ding, Lixin Tian +, Honglin Yang

Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang 212013, P. R. China

(*Received June 18 2005, Accepted July 7 2005*)

Abstract. Energy sustainability is involved to build a game in which a central government and two local governments take into consideration both the economic growth and sustainable development of energy. Then energy strategy selections of the three governments are analyzed. Some results about the optimal allocation of energy for the national economic benefits are given.

Keywords: energy, energy sustainability, sustainable development, game.

1. Introduction

In the literatures, many papers have discussed the problem of allocation of limited energy resources. In these researches, several techniques have been used to deal with such a problem, such as linear and nonlinear programming, dynamic programming, multi-objective optimization, dynamic optimization approach, fuzzy analysis, stochastic approach, etc. Joshi, Bharati et al. (1991) used a simplified linear model to allocate energy among local regions based on the optimized cost function energy supply. Ramanathan, R. and Ganesh, L.S. (1993) presented a multi-objective programming model for allocation of energy resources to various energy end uses. And a multi-objective energy resource allocation model is studied in a fuzzy manner by Agrawal, R.K. and Singh, S.P. (2001). T. Mezher et al. (1998) presented a multi-objective allocation model based upon pre-emptive goal programming techniques. Using stochastic dynamic programming technique, Houmin Yan et al. (2000) studied the problem of energy demand allocation for purchasers. Liu, L. et al. (2000) introduced an integrated dynamic optimization approach for nonrenewable energy resources management under uncertainty.

But in most countries, energy is not distributed equally in each area. For example, the current energy situation in China is that energy in the eastern area is in short supply, but there are abundant energy resources in the western region. By the Develop-the-west Strategy in China, energy development such as Transmission of Electricity from the West to the East and Transferring Natural Gas from the West to the East, is just to utilize the western energy advantage, develop the abundant energy resources in the west, and get the optimum allocation of resources nationwide. But energy development must exercise the sustainable development strategy, not to gain the economic development simply by exhausting the rare energy resources. This paper’s energy allocation is to discuss how the energy shifts from the abundant area to the poor area by a game analysis of decision makers.

In this paper, our economic energy system includes three parts: central government, regions A and B. Region A is in energy shortage; region B has abundant energy reserves but poorer economic development (A is the East and B is the West for the case of China). If some energy resources in region B are developed by the central government and transform to A, on one hand, the economy development of B itself can be enhanced, and on the other hand the economy development of A can also be strengthened since the energy developed is put in A. The Develop-the-west Strategy in China now is a typical example of this model.

* Research was supported by the National Nature Science Foundation of China (No.90210004), National Social Science Foundation of China (No.02BJY051), Social Science Foundation of Jiangsu Province of China (No.04SJD790010) and Nature Science Foundation of Jiangsu High Education System (No.02KJD110003).

+ Corresponding author. Tel.: +86-511-8791467; fax: +86-8791467.
E-mail address: dgzw@ujs.edu.cn; tianlx@ujs.edu.cn; yhl812@ujs.edu.cn

While choosing energy strategy, however, all the central government and local governments must give consideration at the same time to both the economic development and the sustainability of energy development. The key problem in this paper is that, after weighing the economic growth and the sustainable energy development, the central government must make the decision that, in the economic period, in order to make energy resources reach the optimal allocation according to the national economic sustainable development, how much energy should be transformed from region B to region A. On one hand, if a large number of energy resources are developed in region B, this can bring about the great economic development of B and reduce the shortage of energy supply in region A. On the other hand, excessive development will damage the sustainability of energy development. The two regions A and B face the same problem. After the central government's decision, while taking into consideration the sustainability of energy development, they should make the decision how to use their own energy resource. To answer these questions, we establish a game played by the central government and the local governments, and analyze their strategy choices based on the regional and national interests between economic growth and sustainability of energy development.

In the second section below, we give a measuring concept of sustainable energy development at first : Bearing Ability of Energy of Sustainable Development (BAESD); In the third section we construct the game of energy strategy choice played by the central government and the local regions A and B; In the forth section we analyze the response of the local governments A and B to the energy development strategy of the central government; In the fifth section we finally reach the optimum decision of the central government for the national economic benefits.

2. Bearing ability of energy of sustainable development

In order to measure the economic sustainability of energy, we define a variable to describe the usability of energy, according to which the authorities make decision. This variable is denoted by the symbol b, and called Bearing Ability of Energy of Sustainable Development or Bearing Ability of Energy (BAE) simply.

In the energy economy, we use the variable Q to measure the full quantity of the usable energy in one economic period. We can gain the greatest economic development if we exhaust all the energy resource, but from the viewpoint of energy, this does not mean that we can stand on the best economical level, because the exhaustion of energy implies the damage of the sustainability of the economy. If the actual energy input is e, $\gamma = 1 - \frac{e}{Q}$ is then the remainder coefficient of the usable energy resource, $0 \le \gamma \le 1$.In our paper, we suppose first that b is an increasing function of γ.

In addition, there are decision makers in any economy. When they make decisions, they have in mind some personal ideas about the sustainability of the energy resource. Different makers may have different attitudes toward the same economic situation, such as toward the BAE. We use a variable η to describe the attitude of the decision makers toward the BAE, $0 \le \eta \le 1$.

Now the BAE is defined as a function of γ and η, i.e. $b = b(\eta, \gamma)$. The function $b(\eta, \gamma)$ is supposed to satisfy the following conditions:

(B1) $\frac{\partial b}{\partial \gamma} \ge 0$, $\frac{\partial b}{\partial \eta} \le 0$;

(B2) $\frac{\partial^2 b}{\partial \gamma^2} \le 0$, $\frac{\partial^2 b}{\partial \eta^2} \ge 0$, $\frac{\partial^2 b}{\partial \gamma \partial \eta} \ge 0$;

(B3) $b(\eta, 0) = 0$, $b(\eta, 1) = 1$, $b(0, \gamma) = 1$.

From (B1) and (B3), $0 \le b(\eta, \gamma) \le 1$.

For these assumptions, we give our explanations as follows.

The meaning of the first inequality in (B1) is obvious. The second inequality tells us the more the decision makers are concerned about the BAE, the less satisfied they feel with it.

In (B2), the first inequality is the traditional economic assumption, which implies the marginal decreasing contribution of the remainder coefficient of energy to the BAE. For the second inequality, we note that $\frac{\partial b}{\partial \eta}$ is negative and $\frac{\partial^2 b}{\partial \eta^2} \geq 0$ implies $\frac{\partial b}{\partial \eta}$ is an increasing function of η. Thus the absolute value of $\frac{\partial b}{\partial \eta}$ will be smaller if η increases. That is, if the decision makers have already been concerned about the BAE, there would be little alteration of the makers' satisfaction with it when the makers take more care of it. The third inequality tells us $\frac{\partial b}{\partial \gamma}$ is also an increasing function of η, i.e. for the decision makers, the more care is taken of the BAE, the more value is attributed to the marginal contribution of the energy remainder coefficient.

In (B4), the first equation shows that the BAE is zero if all the energy is used up. The second equation implies the BAE gains the maximum value 1 if all the energy is reserved. The third equation tells us, the decision makers who are indifferent to the sustainability of energy development believe that the BAE of their economy is just the same as the initial value 1 which is gained at the initial state when no energy is consumed.

From these economic meanings, we believe the three conditions satisfied by the BAE function $b(\eta,\gamma)$ are rational. Thus the paper below uses the function $b(\eta,\gamma)$ to measure the BAE of our economy when some of its energy is consumed.

3. The game model

Suppose the central government will make decision to develop some quantity of energy E in the region B, and put all of it into A to reduce A's energy shortage. And both the region A and B will make decision to put some of its own energy resource into its own economy. Suppose the self-input of A is e_1, and the self-input of B is e_2. Then the total amount put in A is the sum of central development and A's self-input, i.e. $e_1 + E$, and the total one put in B is just B's self-input, i.e. e_2. The total amount of energy developed in B is the sum of central development in B and B's self-input, i.e. $e_2 + E$, and the total one developed in A is just A's self-input e_1.

We pay our attention to the economy in which energy plays the key role, or in which the other factors but energy are fixed. So the economic growth is decided by the energy input.

Suppose A's production function is f, then its economic growth can be written as $G_1 = f(e_1 + E)$ since the total energy input is $e_1 + E$.

For region B, we can assume that the central development E will bring about some growth of its economy because the central government will invest a great amount of capital into it. Thus B's economic growth is an increasing function of the central development E, and of course an increasing function of B's self-input e_2. We have then the B's growth function $G_2 = g(e_2, E)$.

The growth functions $f(e)$ and $g(e,E)$ of the two regions can be supposed to satisfy:

(F1) $f'(e) \geq 0$,

(F2) $f''(e) \leq 0$,

(G1) $\frac{\partial g}{\partial e} \geq 0$, $\frac{\partial g}{\partial E} \geq 0$,

(G2) $\frac{\partial^2 g}{\partial e^2} \leq 0$, $\frac{\partial^2 g}{\partial E^2} \leq 0$.

All these are common conditions on the production function in economics.

What we have discussed above is only the problem of growth. As is pointed out in the introduction, the decision makers must consider the sustainability of the economic growth. Then we need to take into consideration the BAE discussed in section 2.

By definition in section 2, we get A's BAE

$$b_1 = b(\eta_1, \gamma_1) = b(\eta_1, 1 - \frac{e_1}{Q_1}) \hat{=} b(\eta_1, 1 - r_1 e_1) \tag{1}$$

where Q_1 is the maximum usable energy of A itself in one economic period, $r_1 = \frac{1}{Q_1}$, $r_1 e_1$ is then the energy consumption coefficient of A, and η_1 is the attitude of A's decision makers toward the region's BAE. Taking into consideration both the growth G_1 and the BAE b_1, we define the utility function u_1 of A's sustainable economic development, $u_1 = u_1(G_1, b_1)$.

And the utility function u_2 of B is defined by the same way as u_1, $u_2 = u_2(G_2, b_2)$. The expression of b_2 is in the same form as (1)

$$b_2 = b(\eta_2, \gamma_2) = b(\eta_2, 1 - \frac{e_2 + E}{Q_2}) \hat{=} b(\eta_2, 1 - r_2(e_2 + E)) \tag{2}$$

where Q_2 is the maximum usable energy of A in one economic period, $r_2 = \frac{1}{Q_2}$, $r_2(e_2 + E)$ is the energy consumption coefficient of B, and η_1 the attitude of A's decision makers.

Similarly, the central government will take into consideration both the national economic growth $G = G_1 + G_2$ and the national BAE b,

$$b = b(\eta, \gamma) = b(\eta, 1 - \frac{e_1 + e_2 + E}{Q}) \hat{=} b(\eta, 1 - r(e_1 + e_2 + E)) \tag{3}$$

where $Q = Q_1 + Q_2$ is the national maximum usable energy, $r = \frac{1}{Q}$, $r(e_1 + e_2 + E)$ the national energy consumption coefficient, and η the attitude of the central government. The utility function of the central government is defined by u, $u = u(G, b)$.

These utility functions are all increasing with the economic growth and the BAE b. Moreover, the three functions u_1, u_2 and u can be assumed to satisfy the classical conditions of utility function:

(U1) $\frac{\partial u}{\partial G} \geq 0, \frac{\partial u}{\partial b} \geq 0,$

(U2) $\frac{\partial^2 u}{\partial G^2} \leq 0, \frac{\partial^2 u}{\partial b^2} \leq 0.$

Now the three players game is that, the central government makes decision to develop and transfer some energy E from B to A, and the region A (or B) then makes decision to put its own energy e_1 (or e_2) to its local economy.

We need to get the Nash-equilibrium strategy (E^*, e_1^*, e_2^*) of the central government and the region A and B. To solve this game, we deal with first below the best response of A and B to the central decision.

4. The best response to the central government

Facing the central decision E, regions A and B make decision to maximize their own utility u_1 and u_2. A's problem is to

$$\max_{e_1} \; u_1 = u_1(f(e_1 + E), b(\eta_1, 1 - r_1 e_1)). \tag{4}$$

Solving $\frac{du_1}{de_1} = 0$, we get A's best response equation

$$\frac{\partial u_1}{\partial f} f' - r_1 \frac{\partial u_1}{\partial b} b_2' = 0, \tag{5}$$

where b_2' represents the partial derivative of b with respect to its second variable (the other notations below mean the same.)

The equation (5) determines A's best response function $e_1^* = e_1^*(\eta_1, E)$.

Similarly, B is to solve the maximum problem

$$\max_{e_2} \; u_2 = u_2(g(e_2, E), b(\eta_2, 1 - r_2(e_2 + E)). \tag{6}$$

From $\frac{du_2}{de_2} = 0$, get B's best response equation

$$\frac{\partial u_2}{\partial g} g_1' - r_2 \frac{\partial u_2}{\partial b} b_2' = 0 \tag{7}$$

(7) then determines B's best response function $e_2^* = e_2^*(\eta_2, E)$.

Proposition 1: If the conditions $\frac{\partial^2 u_1}{\partial G \partial b} \ge 0$ and $\frac{\partial^2 u_2}{\partial G \partial b} \ge 0$ hold for the utility functions u_1 and u_2, then e_1^* and e_2^* satisfy

1) $\frac{\partial e_1^*}{\partial E} \le 0$, $\frac{\partial e_1^*}{\partial \eta_1} \le 0$,

2) $\frac{\partial e_2^*}{\partial \eta_2} \le 0$.

Proof: Differentiate the equation (5) with respect to the variable E, then follows

$$[\frac{\partial^2 u_1}{\partial f^2} f'(1 + \frac{\partial e_1}{\partial E}) - r_1 \frac{\partial^2 u_1}{\partial f \partial b} b_2' \frac{\partial e_1}{\partial E}] f' + \frac{\partial u_1}{\partial f} f''(1 + \frac{\partial e_1}{\partial E})$$
$$- r_1 [\frac{\partial^2 u_1}{\partial f \partial b} f'(1 + \frac{\partial e_1}{\partial E}) - r_1 \frac{\partial^2 u_1}{\partial b^2} b_2' \frac{\partial e_1}{\partial E}] b_2' - r_1 \frac{\partial u_1}{\partial b} (-r_1 b_{22}'' \frac{\partial e_1}{\partial E}) = 0$$

and it is simplified to

$$[\frac{\partial^2 u_1}{\partial f^2} f'^2 - 2 r_1 \frac{\partial^2 u_1}{\partial f \partial b} b' f' + r_1^2 \frac{\partial^2 u_1}{\partial b^2} b_2'^2 + \frac{\partial u_1}{\partial f} f'' + r_1^2 \frac{\partial u_1}{\partial b} b_{22}''] \frac{\partial e_1}{\partial E}$$
$$= r_1 \frac{\partial^2 u_1}{\partial f \partial b} b' f' - \frac{\partial^2 u_1}{\partial f^2} f'^2 - \frac{\partial u_1}{\partial f} f''$$

From the assumptions of u_1, f and b, and the condition $\frac{\partial^2 u_1}{\partial f \partial b} \ge 0$, we have that the coefficient of $\frac{\partial e_1}{\partial E}$ on left side is nonnegative and the right side is positive. Thus holds $\frac{\partial e_1^*}{\partial E} \le 0$.

In the same way, we differentiate (7) with respect to η_2. From the assumptions for u_2, g and b, holds $\frac{\partial e_1^*}{\partial \eta_1} \le 0$ and $\frac{\partial e_2^*}{\partial \eta_2} \le 0$. The proof finished.

There are some remarks on this conclusion. First, the condition $\frac{\partial^2 u}{\partial G \partial b} \ge 0$ is satisfied by a large number of utility functions, for instance, by the Cobb-Douglas utility function $u = AG^\alpha b^\beta (\alpha, \beta \ge 0)$. Second, these properties have their economic implications. $\frac{\partial e_1^*}{\partial E} \le 0$ implies that A's self-input e_1 is decreasing with the central development in B. This is very the purpose to reduce the energy shortage of A. $\frac{\partial e_1^*}{\partial \eta_1} \le 0$ and $\frac{\partial e_2^*}{\partial \eta_2} \le 0$ says the more attention paid to the BAE, the less energy input. Third, from our conditions we can't get $\frac{\partial e_2^*}{\partial E} \le 0$. Because the central development will stimulate the economy of region B, the more the central development, the lager the scale of B's economy and the more need of energy input.

5. The best strategy of the central government

For simplicity, we chose specific functions below to analyze the central best decision.

First, the growth functions of A and B are specified to have linear form (a kind of Cobb-Douglas function with one input). That is, $f(e_1+E) = A(e_1+E)$, $g(e_2, E) = B(E)e_2$, where A, $B(E)$ represent the other growth factors such as the technological advances, and can describe in some way the efficiency of input-output of energy. We assume that $B'(E) \ge 0$, $B''(E) \le 0$. Second, suppose the central government, region A and region B have the same attitude toward BAE, and the their functions of BAE are in the same form $b(\eta, r) = r^\eta$, $0 \le \eta, r \le 1$. It is easy to see this function satisfies the conditions (B1) to (B3) assumed to b above. Finally, the utility functions of the three players are assumed to have the product form, i.e. $u_1(G_1, b_1) = G_1 b_1$, $u_2(G_2, b_2) = G_2 b_2$, $u(G, b) = Gb$.

Taking these functions into (5) and (6), we have the best response equations of region A and region B:

$$(1 - r_1 e_1) - \eta r_1 (e_1 + E) = 0 \tag{8}$$

$$(1 - r_2(e_2 + E)) - \eta r_2 e_2 = 0 \,. \tag{9}$$

From (8) and (9), get their best response functions:

$$e_1^* = \frac{1}{(1+\eta) r_1} - \frac{\eta}{(1+\eta)} E \tag{10}$$

$$e_2^* = \frac{1}{(1+\eta) r_2} - \frac{1}{(1+\eta)} E \,.$$

Now the central government is to solve the following maximum program

$$\max_E \ u = [A(e_1^* + E) + B(E)(e_2^*)][1 - r(e_1^* + e_2^* + E)]^\eta \,. \tag{12}$$

Noting that $e_1^* + e_2^* + E$ is a constant independent of the variable E, the program is equivalent to

$$\max_E v = A(e_1^* + E) + B(E)(e_2^*) \,. \tag{13}$$

Let $\frac{dv}{dE} = 0$, then we have

$$A(1 + \frac{\partial e_1^*}{\partial E}) + B'(E) e_2^* + B(E) \frac{\partial e_2^*}{\partial E} = 0 \,. \tag{14}$$

From (10), (11) and (14), get the equation of the central best decision

$$A + B'(E)(\frac{1}{r_2} - E) - B(E) = 0 \,. \tag{15}$$

Because $\frac{1}{r_2} = Q_2$, (15) can be written as

$$A + B'(E)(Q_2 - E) = B(E) \,, \tag{16}$$

(16) determines the central best decision function $E^* = E^*(Q_2, A)$. It has the following property.

Proposition 2: E^* is an increasing function of the variable A, i.e. $\frac{\partial E^*}{\partial A} \geq 0$.

Proof: Differentiate the equation (16) with respect to the variable A, then follows

$$1+[B''(E)(Q_2-E)-B'(E)]\frac{\partial E}{\partial A}=B'(E)\frac{\partial E}{\partial A} \tag{17}$$

It is simplified to

$$[2B'(E)-B''(E)(Q_2-E)]\frac{\partial E}{\partial A}=1 \tag{18}$$

Noting that Q_2 is B's maximum usable energy and E is a part which will be transferred to A, we have $Q_2 > E$. Thus from the assumptions of $B(E)$, the coefficient of $\frac{\partial E}{\partial A}$ on the left side of (18) is nonnegative. We have proved that $\frac{\partial E^*}{\partial A} \geq 0$. The proof finished.

The parameter A represents the efficiency of input-output of energy. Proposition 2 tells us, for the national benefit the central government will develop more energy of B to region A if A use energy more efficiently. So region A should see this and base its economic growth on the high efficient use of energy.

6. Conclusions

In the energy strategy choice game, all the central government and the local governments should take consideration into both the economic growth and sustainability of energy development. Our analysis shows that if the decision makers pay more attention to the sustainable use of energy, then energy resource would be more preserved. Second, the implementations of B's energy develop let the energy move from A to B. From the viewpoint of optimal energy allocation, if A hope to obtain more energy from B to reduce its energy shortage, it should choose the development pattern of efficient energy use.

7. References

[1] R. K. Agrawal, and S.P. Singh, "*Energy allocations for cooking in UP households (India): A fuzzy multi-objective analysis*", Energy Conversion and Management 42(18), pp. 2139-2154, 2001.

[2] D. Fudenberg, J. Tirole, "*Game Theory*", The MIT Press, Cambridge, MA, 1993.

[3] Houmin Yan, Houzhong Yan, and Hanqin Zhang, "*Demand allocation analysis for energy purchasers in deregulated energy markets*", 2000 International Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT 2000), London, pp. 344-348, 2000.

[4] Bharati Joshi, T. S. Bhatti, N. K. Bansal, K. Rijal, and P. D. Grover, "*Decentralized energy planning model for optimum resource allocation with a case study of the domestic sector of rurals in Nepal*", International Journal of Energy Research 15(1), pp. 71-78, 1991.

[5] L. Liu, G. H. Huang, G. A. Fuller, A. Chakma, and H. C. Guo, "*Dynamic optimization approach for nonrenewable energy resources management under uncertainty*", Journal of Petroleum Science & Engineering 26(1), pp. 301-309,2000.

[6] R. Ramanathan, and L. S. Ganesh, "*Multiobjective programming approach to energy resource allocation problems*", International Journal of Energy Research 17(2), pp. 105-119, 1993.

[7] Mezher Toufic, Chedidy Riad, and Zahabi Wissam, "*Energy resource allocation using multi-objective goal programming: the case of Lebanon*", Applied Energy 61, pp. 175-192, 1998.