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A class of random fuzzy programming model and its application to vehicle routing problem*

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Abstract. In this research we concentrate on developing and analyzing a programming model for a version of uncertain vehicle routing problems (VRPs). The customers demands are random fuzzy variables and the travel times between customers follow given probability distributions. Vehicles set out from a single depot, serve a number of customers and upon completion of their service, return to the depot. Each customer whose location is fixed has to be visited by exactly one and only one vehicle and a vehicle will be assigned for only one route. The objective is to minimize the total travel time while satisfying the capacity and arrival time constraints on the greatest degree. Due to the NP-hardness of uncertain VRPs, a pure genetic algorithm (GA) is presented. The proposed model and algorithm, which can be potentially useful in solving uncertain VRPs, provide significant solutions to a medical waste collection VRP in real-life.

Keywords: random fuzzy variable, random fuzzy programming, uncertain VRPs, GA.

1. Introduction

The vehicle routing problem (VRP) is one of the most challenging combinatorial optimization tasks. Defined more than 40 years ago, this problem consists in designing the optimal set of routes for fleet of vehicles in order to serve a given set of customers. The interest in VRPs is motivated by its practical relevance as well as by its considerable difficulty.

In fact, there are uncertain factors in VRPs, such as demands of consumers, travel times between consumers, locations of consumers, number of vehicles, consumers to be visited [29]. So a lot of papers in world literature have been devoted to the uncertain VRPs. Stochastic VRPs (SVRPS), where the uncertain factors in VRPs are assumed to follow given probability distributions, have been widely studied ([16], [2], [3], [18], [24], [13]). SVRPs may be employed to model a number of business situations that arise in the area of distribution. For some example of applications followed, see [2], [17], [8]. Other authors have considered the additional case where the uncertain factors are provided as fuzzy variables. This variation is known as fuzzy VRPs ([28], [31]).There is widespread evidence that the exact values of the mean demands which follow probability distributions are very difficult to be obtained. Therefore, this work deals with a variation of uncertain VRPs where the customers' demands are random fuzzy variables and the travel times between customers are random variables. This situation arises in practice whenever a distribution company faces the problem of collections or deliveries to a set of customers whose demands follow continuous probability distributions and the probability distributions are completely known except for the mean values (start-up operations of a distributing system are just starting up, the records of demand at some nodes that are not up-to-date, etc). In other words, we cannot obtain the deterministic values of some probability density functions' parameters and the information of them is not precise enough. For example, based on experience and records, it can be concluded the mean value of probability density function at a node is “approximately between 1t and 2t'”. In this case the demand is random fuzzy variable.

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The concept of random fuzzy variables was provided by [12], [10], and [30]. By random fuzzy programming we mean the optimization theory in random fuzzy environments. For the applications in different ways of random fuzzy programming in recent years, see [15], [11], and [14]. To the best of our knowledge, there is a little research for the properties of the mathematical programming with random fuzzy coefficients and VRPs with random fuzzy demands. The only related study appears to be the paper [7] which proposed a method to solve a class of model with random fuzzy coefficients in both the objective functions and constraint functions and applied it in the capacitated VRPs (CVRP) with random fuzzy demands. Based on the concept of random fuzzy variables introduced by [12], our objective is to provide workable formulations and exact algorithms for a class of uncertain VRPs with capacity and arrival time constraints, which are common problems in practice.

The rest of the paper is organized as follows. In section 2, we develop a programming model with random fuzzy and random variables for VRPs which consider capacity and arrival time constraints and present a stochastic programming formulation which includes probabilistic constraints. In section 3, we describe the pure GA for the model. Section 4 is devoted to the application of the model and algorithm to a medical waste collection problem in real-life.

2. Problem statement and modelling

2.1. Description and formulation

We assume that there are n customers in the network to be served. Vehicles set out from depot v0, serve a number of customers and upon completion of their service, return to the depot. Each customer whose location is fixed has to be visited by exactly one and only one vehicle and a vehicle will be assigned for only one route. See Fig.1. Travel times are independent random variables $T_{ij} \sim \mathcal{N}(\overline{T}_{ij}, \sigma_i^2)$, $i \neq j$, $i, j = 1, 2, \ldots, n$.
The demands are independent random fuzzy variables $\hat{q}_{i\alpha} \sim \mathcal{N}(\tilde{\rho}_{ij}, \sigma_t^2)$ with the trapezoidal fuzzy number $\tilde{\rho}_i = (\rho_{i1}, \rho_{i2}, \rho_{i3}, \rho_{i4}), i = 1, 2, \cdots n$. The α-cut of $\hat{q}_i$ is $q_{i\alpha} = \{q_i(w) \in \Re \mid \mu_{\tilde{\rho}_i}(\rho_i) \geq \alpha\}, \forall \alpha \in [0,1]$.

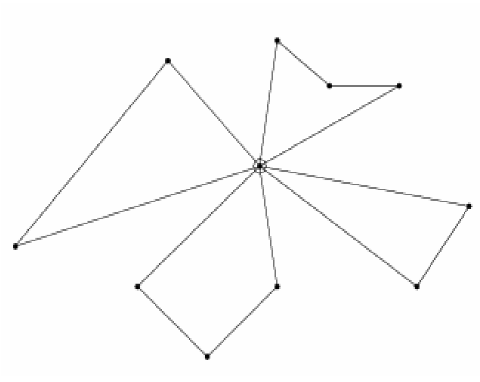

Fig.1: The vehicle route graph

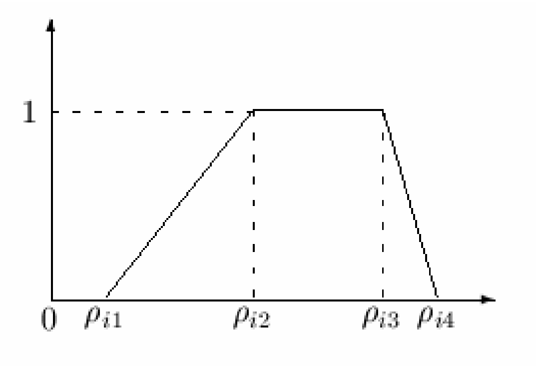

Fig.2: Trapezoidal fuzzy number $\tilde{\rho}_i$

Fig.2 presents the membership function of trapezoidal fuzzy number $\tilde{\rho}_i = (\rho_{i1}, \rho_{i2}, \rho_{i3}, \rho_{i4})$, representing the imprecise mean value at customer i. Obviously, the α-cut of $\tilde{\rho}_i$ is a closed interval, i.e. $\rho_{i\alpha} = [\rho_{i\alpha}^L, \rho_{i\alpha}^R] = [\rho_{i1} + (\rho_{i2} - \rho_{i1})\alpha, \rho_{i4} - (\rho_{i4} - \rho_{i3})\alpha]$, $\forall \alpha \in [0,1]$. The parameters and decision variables used are as follows:

$i = 0$: Depot (v0);

$i = 1, 2, \cdots n$: Customers;

$k = 1, 2, \cdots m$: Vehicles;

Q_k: The physical capacity of vehicle k;

$\hat{q}_i$: The random fuzzy amount of rubbish of plant i;

T_{ij} : The random travel time from plant i to j ;

b_i : The upper time limitation of customer i ;

$AT_i(x,y)$: The arrival time function of some vehicles at customer i ;

$t=(t_1,t_2,\cdots,t_m)$: Each t_k denotes the starting time of vehicle k , $k=1,2,\cdots,m;$

$x=(x_1,x_2,\cdots,x_n)$: Integer decision variables representing n customers with $1\le x_i\le n$ and $x_i\ne x_j$ for all $i\ne j$, $i,j=1,2,\cdots,n;$

$y=(y_1,y_2,\cdots,y_m)$: Integer decision variables with $0\le y_1\le y_2\le\cdots\le y_m\equiv n$.

For each k ($1\le k\le m$), if $y_k=y_{k-1}$, then the vehicle k is not used; if $y_k>y_{k-1}$, then vehicle k is used and starts from the depot at the time t_k , and the route of vehicle k is

$$0\to x_{y_{k-1}+1}\to x_{y_{k-1}+2}\to\cdots\to x_{y_k}\to 0.$$

Let $TT_k(x,y)$ be the total travel time of vehicle $k, k=1,2,\cdots,m.$ Then we have

$$TT_k(x,y)=\begin{cases}T_{0x_{y_{k-1}+1}}+\sum_{j=y_{k-1}+1}^{y_k-1}T_{x_jx_{j+1}}+T_{x_{y_k}0}, & \text{if } y_k>y_{k-1}\\ 0, & \text{if } y_k=y_{k-1}\end{cases}\tag{2.1}$$

for $k=1,2,\cdots,m.$

For each k with $1<k<m$, if $y_k>y_{k-1}$, then we have

$$AT_{x_{y_{k-1}+1}}(x,y)=t_k+T_{0x_{y_{k-1}+1}}\tag{2.2}$$

and

$$AT_{x_{y_{k-1}+i}}(x,y)=AT_{x_{y_{k-1}+i-1}}(x,y)+T_{x_{y_{k-1}+i-1}x_{y_{k-1}+i}}\tag{2.3}$$

for $2\le i\le y_k-y_{k-1}$. Note that the service times are not considered. Each customer i must be visited before the upper time limitation b_i , and then we have the following arrival time constraints,

$$AT_i(x,y)\le b_i, i=1,2,\cdots,n.\tag{2.4}$$

Obviously, it follows that from the randomness of travel time T_{ij}'s $TT_k(x,y)$ and $AT_i(x,y,t)$ are all random variables determined by (2.1), (2.2) and (2.3).

The total demand of consumers for each route may not exceed the capacity of the vehicle which serves that route, that is

$$\sum_{j=y_{k-1}+1}^{y_k}\hat{q}_{x_j}\le Q_k, k=1,2,\cdots,m.\tag{2.5}$$

Let

$$q(w)=(q_1(w),q_2(w),\cdots,q_n(w))$$

and

$$L_\alpha(\hat{q})=\{q(w)\mid q_i(w)\in q_{i\alpha}, i=1,2,\cdots,n\}.$$

If the decision maker makes the routing plan in order to minimize total travel time while satisfy the capacity constraints (2.5) and arrival time constraints (2.4) for a given α , then we have problem (2.6) which is a stochastic programming.

The parameters $q_i(w), i=1,2,\cdots,n$ are not regarded as constant numbers but decision variables. Throughout, we shall focus our attention on the stochastic parametric model (2.6).

$$\min \sum_{s=1}^{m} TT_s(x,y)$$
$$s.t.\begin{cases} \sum_{j=y_{k-1}+1}^{y_k} q_{x_j}(w) \le Q_k, \\ AT_i(x,y) \le b_i, \\ 1 \le x_i \le n, x_i \ne x_j, i \ne j, \\ 0 \le y_1 \le y_2 \le \cdots \le y_m \equiv n, \\ q(w) \in L_\alpha(\hat{q}), \\ x_i, y_j, \text{ integers}, k = 1,2,\cdots,m,\ i, j = 1,2,\cdots,n. \end{cases} \tag{2.6}$$

2.2. Solution framework

Here, we consider the assistant problem of (2.6) as

$$\min E\left[\sum_{s=1}^{m} TT_s(x,y)\right]$$
$$s.t.\begin{cases} \mathrm{P}\left(\sum_{j=y_{k-1}+1}^{y_k} \hat{q}_{x_j} \le Q_k\right) \ge \beta_k, \\ \mathrm{P}\left(AT_i(x,y) \le b_i\right) \ge \gamma_{i,} \\ 1 \le x_i \le n, x_i \ne x_j, i \ne j, \\ 0 \le y_1 \le y_2 \le \cdots \le y_m \equiv n, \\ q(w) \in L_\alpha(\hat{q}), \\ x_i, y_j, \text{ integers}, k = 1,2,\cdots,m,\ i, j = 1,2,\cdots,n. \end{cases} \tag{2.7}$$

where $\mathrm{E}\left[\bullet\right]$ is the expected value operator. Problem (2.7) is the α -chance constrained programming of (2.6). In the following, we show how the transformation proposed in [7] applies to our model. If $y_k > y_{k-1}$, then $\mathrm{E}\left[\sum_{k=1}^{m} \mathrm{TT_k}(x,y)\right] = \sum_{k=1}^{m} \overline{\mathrm{T}}_{0x_{y_{k-1}+1}} + \sum_{j=y_{k-1}+1}^{y_k-1} \overline{T}_{x_j x_{j+1}} + \overline{T}_{x_{y_k}0} = \sum_{k=1}^{m} \overline{TT}_k(x,y)$ for $k = 1,2,\cdots m$. As it is well known in [20], the affine combination of independent, normally distributed random distributed random variables is normally distributed.

For $k = 1,2,\cdots m$, if $y_k > y_{k-1,}$ let Φ is the Laplace function, then

$$\mathrm{P}\left(\sum_{j=y_{k-1}+1}^{y_k} q_{x_j}(w) \le Q_k\right) = \mathrm{P}\left(\frac{\sum_{j=y_{k-1}+1}^{y_k} q_{x_j}(w) - \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j}}{\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2}} \le \frac{Q_k - \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j}}{\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2}}\right) = \Phi\left(\frac{Q_k - \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j}}{\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2}}\right) \ge \beta_k$$

and

$$\sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j} + \Phi^{-1}\left(\beta_k\right)\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2} \le Q_k;\ \text{if}\ y_k = y_{k-1,}\ \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j} + \Phi^{-1}\left(\beta_k\right)\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2} = 0 \le Q_k.$$

Now let

$$q_k(\rho) = \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j} + \Phi^{-1}\left(\beta_k\right)\sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2} \le Q_k, k = 1,2,\cdots m,$$

where $\rho = (\rho_1, \rho_2, \cdots, \rho_n)$ and we introduce the set-valued function

$$S_k(\rho) = \left\{ \left(x_k, y_k \right) \mid q_k(\rho) \le Q_k \right\}.$$

For $\forall (x_k, y_k) \in S_k(\rho^2)$, we have $q_k(\rho^2) \le Q_k$. If

$$\rho^1 \le \rho^2,\ q_k(\rho^1) \le q_k(\rho^2) \le Q_k.$$

Hence,

$$(x_k, y_k) \in S_k(\rho^1) \text{ and } S_k(\rho^1) \supseteq S_k(\rho^2).$$

We can obtain an optimal solution to (2.7) by solving the following problem consequently.

$$\min\ E\left[\sum_{s=1}^{m} TT_s(x, y) \right]$$

$$\text{s.t.} \begin{cases} \sum_{j=y_{k-1}+1}^{y_k} \rho_{x_j \alpha}^L + \Phi^{-1}\left(\beta_k\right) \sqrt{\sum_{j=y_{k-1}+1}^{y_k} \sigma_{x_j}^2} \le Q_k, \\ \mathrm{P}\left(AT_i(x, y) \le b_i \right) \ge \gamma_i, \\ 1 \le x_i \le n, x_i \ne x_j, i \ne j, \\ 0 \le y_1 \le y_2 \le \cdots \le y_m \equiv n, \\ x_i, y_j,\ \text{integers},\ k = 1, 2, \cdots, m,\ i, j = 1, 2, \cdots, n \end{cases} \tag{2.8}$$

This reformulation allows solving a unique stochastic problem, even if of larger size, rather than solving a family of stochastic programming problems.

3. Algorithms

Genetic algorithms (GAs) have been widespread applied to various combinatorial optimization problems, including certain types of vehicle routing problem, especially where time windows are included (see [22], [1], [21]).

The study of them demonstrated that GAs is an effective approach to solving the basic VRP. Here we use the pure GA to solve the model (2.8) we proposed. The steps of the pure GA can be summarized as follows.

Step1. Initialize chromosomes at random whose feasibility are checked by the constraints of (2.8).

Step2. Update the chromosomes by crossover operations.

Step3. Update the chromosomes by mutation operations.

Step4. Calculate the objective value of each chromosome as its fitness.

Step5. Select the chromosomes.

Step6. Repeat the second to fifth steps for a given number of cycles.

Setp7. Repeat the best chromosome as the optimal solution.

3.1. Initialization Process

Step1.1. Set $pop = 1$ and define $pop_{\max}$ (integer number) as the number of chromosomes.

Step1.2. Generate randomly a population $\left(x, y \right)$ satisfying the capacity constraints.

$$\text{Step1.3.} \begin{cases} \text{If } \mathrm{P}\left(AT_i(x, y) \le b_i \right) \ge \gamma_i, i = 1, 2, \cdots, n, \\ \quad \text{If } pop < pop_{\max}, \\ \quad\quad pop = pop + 1; \text{Goto Step1.2.} \\ \quad \text{Else stop Step1.} \\ \text{Else goto Step1.2.} \end{cases}$$

3.2. Crossover operation

Our crossover to $(x, y), n = 10, m = 4$ is illustrated as an example. Assume there are two parents P1, P2.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	y_1	y_2	y_3	y_4
P1:	4	3	9	5	2	10	8	1	7	6	3	6	9	10
P2:	6	1	5	9	8	7	10	2	4	3	1	5	10	10

Crossover points have been generated between customers 3 and 9, vehicle 1 and 3. Application of our crossover yields the following child solution.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	y_1	y_2	y_3	y_4
C1:	4	3	7	1	8	10	2	5	9	6	1	4	7	10
C2:	6	1	5	9	8	7	10	2	4	3	1	5	10	10

Note that y_1 and y_3 of child 1 are generated randomly. The procedure of this proposed approach is listed below.

Step2.1. Set $index = 1$ and $index_{\max}$ = the number of parents

Step2.2. Let parent=thisPopulation{parents(*index*)}.

Step2.3. Update the chromosomes by crossover operation.

$$\text{Step2.4} \begin{cases} \text{If child satisfy the capacity and arrival time costraints,} \\ \quad \text{If } index < index_{\max}/2, \\ \quad\quad index = index + 2;\ \text{Goto Step 2.3.} \\ \quad \text{Else stop Step2.} \\ \text{Else goto Step 2.3.} \end{cases}$$

3.3. Mutation operation

The objective of the mutation is to disrupt the current chromosome slightly by inserting a new gene. In this research we select two positions at random and then swap on these positions. For instance, from the P1 we generate tow positions, x_4 and x_8, and interchanging x_4 and x_8 will alter a certain number of genes from on parent to produce offspring.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	y_1	y_2	y_3	y_4
P1:	4	3	9	5	2	10	8	1	7	6	3	6	9	10
C1:	4	3	9	1	2	10	8	5	7	6	3	6	9	10

Step3.1. Set $index = 1$ and $index_{\max}$ = the number of parents.

Step3.2. Let parent=thisPopulation{parents(*index*)}.

Step3.3. Update the chromosomes by mutation operation.

$$\text{Step3.4.} \begin{cases} \text{If child satisfy the capacity and arrival time costraints,} \\ \quad \text{If } index < index_{\max}/2, \\ \quad\quad index = index + 1;\ \text{Goto Step 3.3.} \\ \quad \text{Else stop Step3.} \\ \text{Else goto Step 3.3.} \end{cases}$$

4. Applications to medical waste collection

For illustrating the purpose of proposed model and algorithm, we consider a medical waste collection VRP introduced by Fig.3.

One agency has to provide a service for 239 hospitals and clinics shown in Fig.3.Since the beds of some hospitals and clinics have been increased, mean values of daily waste of them are estimated as trapezoidal fuzzy numbers based on the daily waste per day per bed, the utility rate of bed and the everyday patients of clinics. We were to design a routing plan to minimize the total travel time of the vehicles subject to capacity and arrival time constraints. In real-life routing, data management plays a major role in the efficient functioning of a distribution system and this becomes more substantial when the distribution takes place within a detailed road network. In order to produce an efficient routing plan, apart from software that will implement the routing algorithm, information must be collected about:

- The quantity of daily waste per bed or patient, the utility rate of bed and the everyday patients of clinics.
- The quantity records of daily waste that has to be collected from each hospital or clinic.
- The capacity of the vehicles used and the arrival time limitations of hospitals and clinics.
- The spatial characteristics of the road segments examined.
- The speed of the vehicles and considering the spatial characteristics of the roads and the surrounding area.

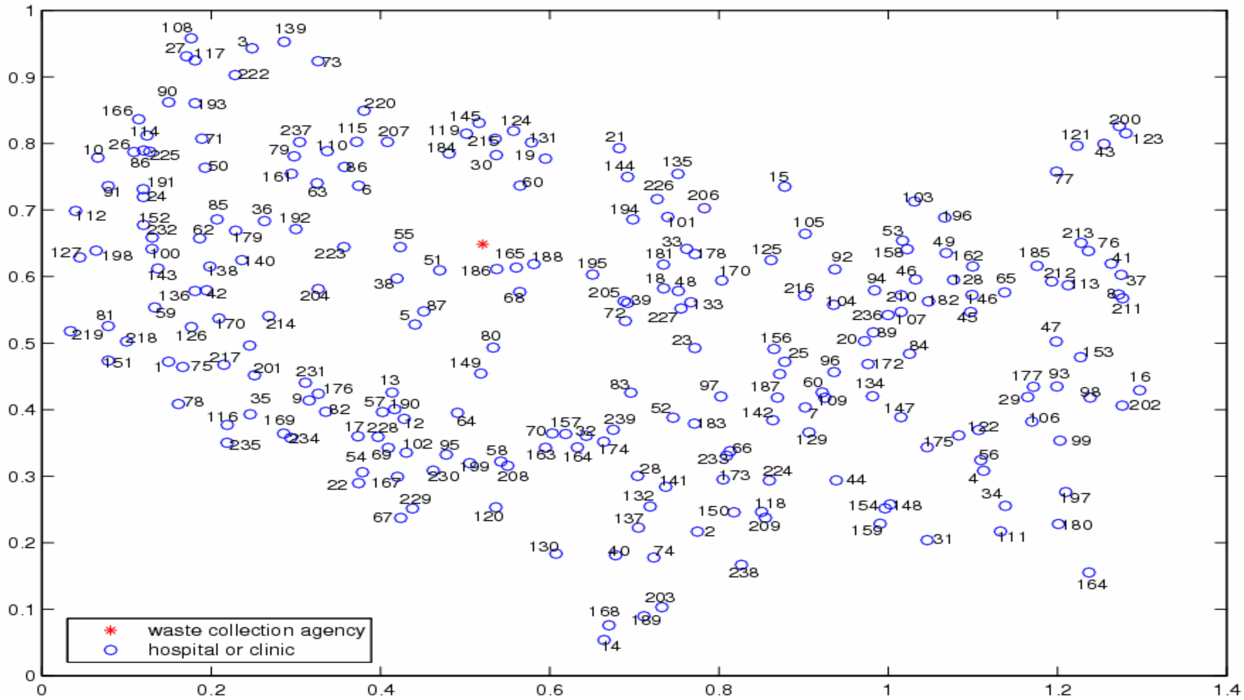

Fig.3: The distribution graph of hospitals and clinics

One approximate solution of this case study are shown in Table 1 under difference confidence levels, providing the information on the customers assigned to the depot, the vehicle routes and the total travel time of all routes developed. No exact routing data are presented for proprietary reasons of the agency. More alternatives can be generated by varying the values of α and $\beta_k, \gamma_k, k = 1, 2, \cdots, m$.

5. Conclusions

The paper has dealt with a version of uncertain VRPs arising in a number of practical situations. The customers' demands are random fuzzy variables and the travel times between customers follow given probability distributions.

We developed and analyzed a new random fuzzy programming model where the target is to minimize the total travel time while satisfying the capacity and arrival time constraints. The model can be solved by means of a pure GA. At last, a medical waste collection VRP in real-life is solved by our method as illustration. Because there is widespread evidence that the exact values of the mean demands are very difficult to obtained, the model and the algorithm in this paper may be interesting and important in the reality. Further applications can be devoted to other uncertain VRP variants in random fuzzy environment and the improving of algorithms for the relative problems.

Table 1: approximate solution of the medical waste collection VRP

239 consumers-13 vehicles																		
Vehicle	Sequence of customers																	
1	124	215	63	71	193	27	119	61	28	173	66	172	93	16	202	162	196	49
	236	118	209	2	189	168	40	57										
2	30	131	178	156	25	221	157	239	32	163	14	203	238	148	154	34	197	122
	142	224	96	18	19													
3	170	101	194	53	41	211	182	134	94	60	109	132	70	149	229	235	169	87
4	133	205	171	198	112	179	85	42	155	35	95	190	5	80	195	144	226	188
5	83	183	97	187	20	89	175	129	7	233	137	199	54	22	201	231	234	217
	75	223	68	164														
6	3	73	145	33	105	104	210	45	11	99	23	227	39	82	9	176	38	214
	204	191	222	237	166	218	81											
7	125	200	123	121	43	65	185	77	37	76	213	128	159	150				
8	192	225	114	50	100	219	127	36	51	165	72	44	31	161	180	111	106	147
	216																	
9	13	17	228	230	69	12	136	143	24	10	26	86	91	152	140	160	135	206
	21																	
10	64	58	208	120	130	74	141	174	52	186								
11	181	103	15	46	146	98	212	4	56	29	177	47	113	8	153	158	84	107
	92	48																
12	184	115	220	207	88	110	79	139	117	108	90	232	126	151	1			
13	167	67	102	116	78	59	138	62	6	55								
Total travel time=377.4minutes, generations=3015										$\alpha = 0.8, \beta_k = 0.9, \gamma_k = 0.8, k = 1, 2, \cdots, m$								

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